CAS for Mathematics E-Learning - a European Perspective

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Key words: *Educational technology, Mathematics, Blended learning, CAS, User Interface.*

Abstract:

We report on activities and progress in a European (Comenius 2.1) project "LTM-Learning Tools for Mathematics" that creates multilingual learning units on the basis of Mathematica and M@th Desktop. A customized user-interface based on palettes is optimized for use in class, and notebooks provide associated math content with a strong emphasis on visualization. The multilingual nature of the learning units makes them useful for transnational school collaboration projects, as has been demonstrated in a pilot project. The problems and exercises are oriented towards applications and encourage explorative group projects. This leads, in particular, to an improved image of mathematics and math instruction among students.

1 Introduction - The LTM Project

While classical ex-cathedra teaching is still the most widespread form of math-instruction in high schools, there is an increasing percentage of teachers who regard computer-supported ways of teaching a welcome means of arousing interest in students and increasing their level of attention. Best results can probably be obtained by combining the use of educational software such as CAS (computer-algebra systems) or DGS (dynamic geometry software) with the opportunities for communication and research offered by web technologies. Acquisition of didactical know-how and exchange of experiences on an European level will eventually lead to an institutionalisation of computer-supported math education in high schools.

Here we report on some activities in the framework of a Comenius 2.1 project named "LTM – Learning Tools for Mathematics" [1], which is a collaboration of three universities and high schools from eight European countries. It is the project's goal to develop multilingual courseware together with didactical concepts for its use in the classroom and material for in-service training of teachers.

The LTM learning units are an extension of the M@th Desktop application written by Reinhard Simonovits and distributed commercially [2]. M@th Desktop is based on the CAS *Mathematica*. There will be a freely available LTM-version of M@th Desktop, but *Mathematica* is still needed to use it. With *Mathematica* as a basis, it is possible to develop interactive and adaptive teaching material in a very time- and cost-effective way, as compared to other (web-based) technologies like Java or Flash. A big disadvantage of this approach is that high licensing fees for *Mathematica* have to be paid by schools or students, which can sometimes only be done in the framework of supported projects.

The LTM project advances a method of blended learning that represents a well-balanced mixture of traditional and computer-aided forms of instruction. Its choice of examples emphasizes applied mathematics and mathematical modelling, which is possible because more tedious computations can be left to the computer. A more explorative approach towards mathematics with group-based learning activities is within reach. A series of pilot projects accompany the LTM project in order to investigate the organisational and practical difficulties of its usage in class, teacher's and students' contentedness, and impact on the level of math skills.

Moreover, LTM aims at facilitating collaboration projects involving classes from different European countries. M@th Desktop will therefore be used in some transnational projects that are currently under preparation and that combine CAS-supported instruction with modern communication tools in order to achieve their results. One of these school collaboration projects has been carried through in 2006 and will be described here in more detail.

In the following we briefly explain the principles and the design of the M@th Desktop and describe some experiences made in classrooms. M@th Desktop supports a variety of different teaching methods, and sometimes this means a radical deviation from traditional habits whose chances and risks have to be explored carefully. We discuss, in particular, how the replacement of purely traditional teaching by computer-assisted forms of instruction might change the role of mathematics in the curriculum and the image of mathematics and math instruction among students.

2 Teaching with M@th Desktop

2.1 The concept of LTM and M@th Desktop

Math software, in particular CAS, usually has a steep learning curve and it demands high expertise from a teacher who wants to make reasonable use of it in a classroom. Many difficulties stem from the necessity to teach mathematical topics and usage of a CAS at the same time. M@th Desktop tries to address these problems and makes every effort to minimize the threshold for using CAS-support for teaching. *Mathematica* can now be used even in situations where this was considered impossible either due to limited time or due to the daunting complexities of a high-end CAS. M@th Desktop achieves this goal by providing *Mathematica* with a simplified user-interface that has been optimized in view of its usage in a classroom. When the project started, only *Mathematica* among the available CAS allowed the manipulation of the user-interface (the "frontend") to the extent necessary for the design of M@th Desktop.

The most important simplification of *Mathematica*'s user interface is achieved by a consequent usage of palettes. Palettes are floating windows containing buttons allowing the teacher or student to execute the required instructions with one click. M@th Desktop contains some general-purpose palettes and some that are designed for special topic areas (see Fig.1). LTM focuses on elementary functions and linear algebra, whereas the commercial version M@th Desktop contains palettes for essentially all topics of math in secondary education.

Fig. 1: Some LTM palettes in *Mathematica*.

A button on a palette typically pastes the template of a *Mathematica* command at the insertion point in the current notebook. For example, the button "Plot" (on the Plot palette) generates the following character string

MDPlot[{ □ , {x , □ , □ }];

Actually, this is just *Mathematica*'s built-in "Plot" command with some minor adaptations. The placeholders can be navigated with the tab-key and filled either with the help of other palette buttons or by typing. Other buttons try to simplify user input in more sophisticated ways, e.g., the Epilog-button pastes a *Mathematica* option (a command-modifier) automatically at the correct position into the line above, without requiring the user to adjust the curser position first:

MDPlot[{ □ , {x , □ , □ }, Epilog → {Red, PointSize[0.025], Point[{□ , □}]}];

After filling the place holders, e.g., in the following way,

MDPlot[{Sin[x], {x,-π,π}, Epilog → {Red, PointSize[0.025], Point[{π/2, 1}]}];

we obtain the graph in Fig.2 immediately after hitting the "Evaluate" button.

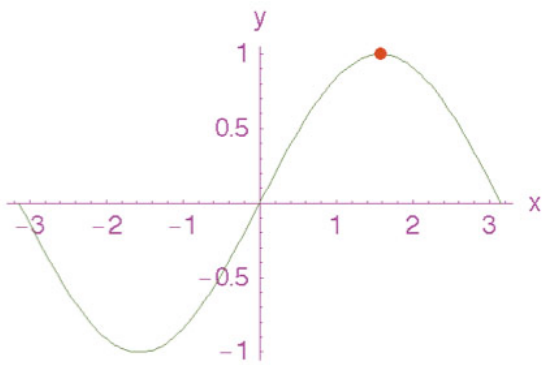

Fig. 2: Output of a LTM Plot-command.

This simplified input method obviously eliminates typing errors, which otherwise virtually can't be avoided due to the rather complicated *Mathematica* syntax. In a classroom, the elimination of these typos is otherwise be a very time-consuming task, and would distract from the actual teaching goals. Among teachers working with M@th Desktop, the simplified input is therefore the perhaps most welcome feature of the software.

The palettes do not hide the syntax from the user, but they allow the user to perform a wide range of tasks without the necessity to learn this syntax. In that way, the teacher can start using the CAS without long preparations. Actually, *Mathematica* has some built-in palettes for character input, and commercial software like *Mathematica* CalcCenter also uses palettes as the main user-interface element.

It might be regarded a disadvantage that a deeper knowledge of the CAS is not required by M@th Desktop. But usually, this is not the goal of teaching Mathematics. Instead, palettes allow us to perform algebraic manipulations and produce complex visualizations, which will certainly help to discuss the mathematical concepts and ideas.

The palettes associated with special math content (like exponential growth) contain - similar to a specialized pocket calculator - all commands and programs that are needed for experimenting and problem solving within that specialized topic area. Many thousands of lines of *Mathematica* program code in packages work in the background to provide the functionality of these buttons. This concentration on individual topics areas is in fact another feature facilitating CAS usage, because only a very limited set of activities has to be mastered in order to use the system successfully within a particular chapter of high-school math.

All palettes devoted to particular topic areas are linked with a *Mathematica* work sheet (notebook) that provides contents for teaching and learning. A typical example of the worksheet's outline is shown in Fig.3. It contains a movie or activity that introduces the learner to the topic at hand, a section with basic definitions, some solved examples that explain the usage of the associated palette, and a lot of exercises, including open-ended tasks and suggestions for group projects.

The M@th Desktop learning units are highly adaptable. They can be changed both by the teacher and by the student in order to create customized learning aids. M@th Desktop offers many opportunities to create own material like exercise sheets, tutorials, test assignments, or project documentations. It provides more skilful users with templates for *Mathematica* packages and even makes it fairly easy to create one’s own palettes. High adaptability of the contents to individual needs and quick development cycles also facilitate the implementation of educational standards, whose inclusion in the curriculum framework is currently undertaken in several countries in the EU. Within the LTM project, we use that adaptability to create versions in several languages that take into account country-specific curricula.

Fig. 3: A typical learning unit with associated palette.

2.2 Learning and teaching scenarios

The concept based on palettes and notebooks is flexible enough to support a great variety of teaching styles - from teacher-led instruction to student-centred explorations. However, certain considerations make us think that the teaching approach with which M@th Desktop would be most compatible, is a mix of computer-supported and traditional phases of teaching (blended learning). The learning tools should be used in more than one way (hands-on sessions, teacher-led activities, problem-solving, project work, etc), and they should be used together with other resources. M@th Desktop is not meant as a stand-alone, self-study resource. In our approach, the computer does not try to supplant the teacher, traditional teaching is still there, and decisions about teaching should be made by the teacher. The computer supports his teaching with palettes, notebooks, visualization and exploration tools, but the teacher neither has to follow a particular trajectory, nor should he give up traditional ways of explaining mathematical concepts.

Evaluation of pilot projects has shown that the highest level of student satisfaction can be reached, if the teacher spends about 50% of his time explaining concepts in the traditional way (i.e., on the blackboard), and let students do simple examples by hand. These traditional phases are highly appreciated by students. Having mastered the basics, they enjoy using M@th Desktop and to check their results and to tackle more demanding problems. They can appreciate the power of the CAS only when they have struggled to do similar tasks by themselves. The computer can then be used any time to produce visualizations, perform experiments or computations, and in order to treat more realistic examples. The teacher changes between traditional and computer-supported teaching as he sees fit. We think that computer-supported teaching methods cannot be used to save time in class. But the time-consuming parallel use of conventional and computer-supported teaching has enormous benefits, as it enables the teacher to present the same ideas repeatedly in two different ways and helps students to acquire knowledge with completely different activities. Thus, mathematical concepts are actually learned twice.

Earlier pilot projects and evaluations of M@th Desktop by teachers have shown that very good results can be obtained if certain basic conditions are fulfilled. Fig.4, for example, shows that students reacted in a very positive way on the usage of M@th Desktop in their class (evaluation of a pilot project in Sibiu/Romania by Pica Elisabeta Ana and Marcu Alexandru, Questionnaire report on usage of Mathematica and M@th Desktop in class).

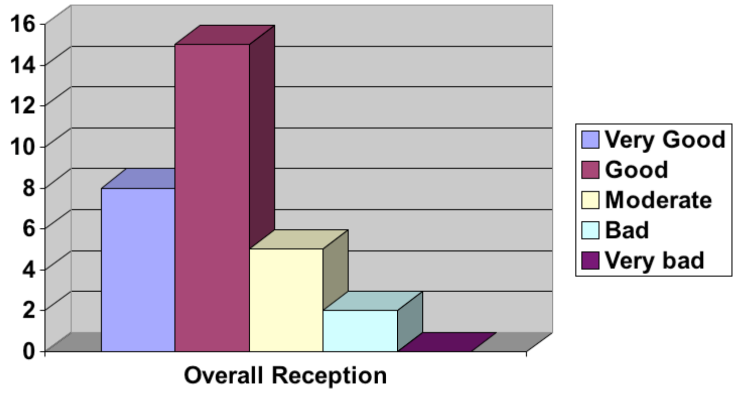

Fig 4. Overall reception of M@th Desktop in class

Good results and a high-level of contentness can only be obtained if certain basic conditions are fulfilled. Just a few hours in a computer lab are certainly not sufficient. There must be enough time and every student should have access to the software also at home. Only then, use of technology can become automatic and thinking about mathematics can play the dominant role. This is, in particular, confirmed by the evaluation of some Austrian pilot projects [4,5,6], where students' attitude and success was particularly positive in the case of notebook classes, but rather negative, if they were only exposed occasionally to M@th Desktop in computer labs.

2.3 Risks of CAS-supported teaching

Use of educational software aims at increasing the quality of math education and the competencies of the teachers. Furthermore, we hope to improve the students' e-learning competencies, which will be useful for their studying at a university and for their later professional life.

But while some competencies might improve in computer-assisted education, other skills will be lost. Certainly, conventional computational and algorithmic skills will diminish, simply because less time is spent with these traditional activities. The teaching community will have to decide whether these changes are, as a whole, positive, and whether they are worth the additional efforts necessary to implement computer-aided education. What are the particular chances and risks of using M@th Desktop?

Teacher-led instruction with computer support often involves the teacher showing something on a computer screen using video projection. Our pilot projects have shown that then at least some students will not be able to follow the lessons, no matter how carefully and slowly the teacher proceeds. During our observations it always happened that at some time some student was not be able to reproduce the teacher's screen content exactly. When that happens, the student will anxiously try to get the seemingly missing information from neighbours or by trial and error. At this moment they stop thinking about mathematics at all, because it is now the technology that requires their full attention. Despite all simplifications that M@th Desktop provides for CAS input, the teacher has to be very careful that all students have reached the same state before any new thoughts are presented.

The trick of having good students help their weaker colleagues relieves the teacher but leads to a mixed success. While the good students actively recapitulate and verbalize their knowledge, the weak students are again in the role of passive recipients. This mechanism is probably responsible for the observation that use of M@th Desktop sometimes increases the gap between good and weak students. Some of the teachers involved in pilot projects therefore suggest that teaching with M@th Desktop should differentiate between good and weak students and they require additional training units for weak students.

M@th Desktop's approach to problem solving basically involves to press buttons in a certain order, i.e., to choose from a finite number of pre-fabricated *Mathematica* commands to which the user provides the appropriate parameters. This requires the student to decompose the problem into suitable sub-tasks that can be accomplished with the provided buttons. In most cases, this removes the algorithmic and computational aspects of problem solving, which is certainly a very radical deviation from traditional approaches. Using M@th Desktop as the only teaching method is probably not the right way, as emphasized in the previous section. It depends on the teacher how the new economy of time in problem solving is used in a responsible way to emphasize thinking about mathematical concepts.

A prolonged brainstorming phase should stand at the beginning of any task and the students should be required to describe their results and the methods used for the solution verbally (M@th Desktop has an "Answer" button for providing specially formatted text cells). Students who cannot speak about the problem and describe its solution correctly most likely do not understand the mathematics behind it. It is usually regarded as a particular advantage of a CAS-based approach to teaching, that graphical, symbolical, numerical, and textual representations of mathematical facts can be easily generated and appear as a unity to the learner. But both teachers and students have to be made aware of these possibilities.

Given the fact that a high-end CAS like *Mathematica* in combination with M@th Desktop is powerful enough to solve all math problems at high-school level with very few keystrokes, many traditional exercises will become rather trivial and pointless. Therefore we try to select examples that illustrate interesting applications of Mathematics to the "real world". A large class of exercises that are now possible even on an elementary level involves fitting data and representing measured phenomena and observations by mathematical expressions.

One particular danger that arises in some situations with the usage of M@th Desktop palettes for problem solving is what we would like to call the "proof-by-exhaustion mannerism". As performing a particular step in a computation is neither particularly laborious nor does it cost much time, students sometimes have the idea just to try out everything by brute force, hoping that the right answer will turn up sooner or later.

We had occasion to observe such mannerism during a project where students had to fit a mathematical function to measured data. The idea was that by analyzing the data plots and by identifying the mechanisms producing the data, the students should judge which function (linear, quadratic, exponential, etc.) would describe the phenomenon best, and then use the method of least squares to adjust some model parameters. Actually, they didn't care at all about the shape of the curves or the mechanisms behind, they just tried *every* model that was at their disposal and chose the one that gave the smallest sum of squared errors. Thus they were very successful in identifying the right model, but they did not think about the underlying problem or the properties of elementary functions. It is, of course, the responsibility of the teacher to identify a problem like this by further tests or by requiring a verbal description of the solution method from the student.

2.4 Active involvement of students - a transnational school project

Projects with students are a particular strength of computer-supported education [3]. M@th Desktop makes it easier to perform application-oriented math projects in a class. Common features of the student projects done in the framework of the LTM project are the following: At the beginning, pupils are asked, whether they want to participate in a group project. Nobody is forced to participate against his/her will. Then there is a common discussion about the project's goals and the guidelines for the evaluation and the assessment of the results, in order to improve the pupil's ability to work in a team. The project will then become part of the teaching activities. The teacher encourages group and team work, and takes special care of improving the performance of pupils during presentations. On some occasions, students are given the opportunity to present their work in front of a larger audience, either at school or at international meetings. It is an important goal to have both sexes participate equally in projects and project presentations.

In the schools of some LTM project partners, M@th Desktop is well established for teaching and as a tool for students' own explorations. In this situation it suggested itself to try a transnational coorperation project with students. In the future, localization of the M@th Desktop software to several languages will even facilitate this type of projects.

In 2006, a transnational pilot project was carried out by HAK Grazbachgasse, Graz, and Lycée René Cassin in Strasbourg [4]. It was agreed that various aspects of the growth of regional energy consumption would be the theme of the collaboration. Regarding the content of the projects, both classes were prepared using the same instructional materials, and the relevant themes were differential calculus and the fitting of curves to given numerical data. This joint project also promised the interesting opportunity to connect mathematics instruction with foreign language instruction and the imparting of “soft skills” such as communications and organizational ability.

The pupils in Graz formed 9 groups of two and, on their own, were to seek the collaboration of 9 groups of three in France. The pupils from Graz gathered data for the Austrian province of Styria, and those from Strasbourg gathered corresponding data for the French region of Alsace. The data were then exchanged, the pupils from Graz analyzed the Alsace data and the pupils from Strasbourg analyzed the data from Styria with the help of M@th Desktop palettes and worksheets. Both results were then communicated to the partner teams, compared, interpreted, described in a *M@th Desktop* notebook, and presented to the class. The notebooks with the results typically contained a description of the assignment, the raw data displayed both in lists and graphically, a description and evaluation of the tested mathematical model, a short description of the results of the pupils from the other country, an interpretive summary with graphical representation, and a prognosis for the year 2005 (for which no data was available at the time).

The project was evaluated by questionnaires and in discussions with the students and the evaluation findings were published in the final report [4,5]. For the Austrian side, the results can be summarized as follows: Students were highly satisfied with mathematics instruction on the basis of M@th Desktop and with the project in particular. They carried out the project with great motivation and managed to retain their great excitement through to the end. A good relationship between teachers and pupils and a positive fundamental class attitude toward mathematics represent possible prerequisites for the success of such projects.

2.5 Changed image of mathematics

In several countries of Europe, evidence of a problem with high-school mathematics seems to be mounting. Recent PISA studies confirmed evidence of a problem with high-school mathematics in many European countries [6]. Students tend to find mathematics less enjoyable and less relevant, and most of them feel that their understanding of the subject is incomplete. This is reflected by the fact that less students choose to study math or science at universities, and those who do have increasing difficulties with the transition between school and university. In the UK, these statements were even confirmed by rigorous investigations [7,8].

It is not clear whether the acquisition of fundamental mathematical ideas and the mastery of traditional skills can be improved through computer-supported, project-oriented instruction. On the contrary, the learners actually spend less time practising these skills than during conventional math education, and in group projects, social skills seem to be at least as important as mathematical skills. It might be the case that students' success in acquiring knowledge and problem solving skills depends to a higher degree on the personality and the commitment of the teacher than on the applied teaching method.

It may, however, be assumed that computer-supported instruction, application-oriented problem solving, and interesting group projects will lead to a much better reputation of mathematics than in a more traditional class with the normal elements of calculation drills. As demonstrated in pilot projects with notebook classes, teaching with M@th Desktop will lead to a very positive image of mathematics as an interesting and relevant field of science. Students usually enjoy these math lessons – it is not a subject to be feared. They no longer question the usefulness and importance of mathematics in the modern world [4]. The choice of application-oriented exercises in the M@th Desktop worksheets and the perceptible power of the CAS certainly contribute to the new positive image of mathematics.

3 Summary and Perspective

The software M@th Desktop encourages students to use symbolical, numerical, graphical and textual forms of representation in problem solving in a way that is balanced and appropriate for the particular purpose at hand. As a result, the student's level of attention as well as their satisfaction with math instruction increases. On the downside, there are sometimes considerable organisational and fiscal problems associated with the use of *Mathematica* and M@th Desktop in class, and optimal conditions can only be found in notebook classes.

Availability of instructional material in several languages, as provided by the LTM project, will facilitate the execution of transnational school projects, where math problems are solved in cooperation between students from different countries. Computer support lets us choose interesting problems with a realistic background, because the more tedious numerical computations may be left to the computer.

Usage of a CAS in education will make much of the algorithmic and computational work found in traditional scenarios obsolete. What then will replace these skills in the future? What role will Mathematics play in future education? Presently, mathematics is still widely associated with numeracy – with counting and calculating. When typical student activities are changed, the future image of Mathematics will become different. It will be a major task of math education to emphasize the role of mathematics in general education, in social discourse, in natural sciences, and as a tool for analyzing economical, social, and scientific matters. In short, mathematics should be presented as an open system rather than a closed intellectual game.

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