

# **Review: Inspecting Everyday Mathematics: Reexamining Culture-Cognition Relations**

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Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life by Jean Lave Culture and Cognitive Development: Studies in Mathematical Understanding by Geoffrey B. Saxe

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# Inspecting Everyday Mathematics: Reexamining Culture-Cognition Relations

Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life. Jean Lave. New York: Cambridge University Press, 1988, 214 pp., \$49.50 hardback, \$14.95 paper. ISBN 0-521-35015-8 hardback, 0-521-35734-9, paper.

Culture and Cognitive Development: Studies in Mathematical Understanding. Geoffrey B. Saxe. Hillsdale, NJ: Lawrence Erlbaum Press, 1990, 236 pp., \$29.95 (tentative). ISBN 0-8058-0273-8.

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his is an exciting time for researchers studying mathematics, learning, and education. The problems of this field have attracted some of the best scholars and scientists in disciplines as diverse as artificial intelligence, cognitive anthropology, cognitive science, developmental psychology, crosscultural psychology, epistemological studies, and interactive learning environment design. These two books make seminal contributions to our understanding of the relations between forms of everyday cultural practices and cognition. Some may be surprised that this description of the books' achievements does not mention mathematics, curriculum, or educational settings, but the focal problems of learning in mathematics provide excellent case studies for what are demonstrably the more fundamental issues of mind-society relations.

The unique contribution of these authors lies in their very careful attention to specific problems surrounding how to account for the occurrence and organization of activities of everyday cultural practices that involve the use of mathematics. In Lave's studies of adults, these arenas of practice are those in which arithmetic comes to be used for best-buy grocery shopping and for home Weight Watcher mathematics measurements. In Saxe's research, the

mathematical practices are those involved in Brazilian child candy sellers' activities, which include wholesale candy purchases, wholesale-retail price translations for setting candy prices, adjustments for the rampant Brazilian inflation, currency change-making from street sales, negotiating prices with customers and wholesale clerks, and competing with other sellers. Both researchers provide a ripe set of methods and conceptual perspectives for guiding research on thinking-in-context, using an illuminating complementarity of ethnographic, structured interview, "simulation experiments," and quantitative methods.

Although Lave and Saxe share the objective of providing more intimate empirical links between cultural practices and cognition for mathematical activities, they could hardly differ more in theoretical orientation. Saxe seeks to understand the interplay between sociohistorical and cognitive developmental processes by providing research that will reveal universal processes in development, while nonetheless he articulates the distinctiveness of cognitive development in different cultural settings, which may differ in their artifacts and forms of social organization. Lave is less concerned with development and more with "connections between cognitive theory, educational forms, and everyday practice" (p. xiii). Lave's more inclusive enterprise is to provide a social anthropology of cognition, which she develops as a theory of cultural practices that is a part of a more encompassing theory of the social order (p. 171). Whereas Saxe want to define and examine a new level of analysis, of activity in sociohistorical context, at which learner participation in cultural practices may lead to the development of cognitive forms and functions, Lave wishes to develop a "general dialectical theory of social order of which practice is a part" (p. 145).

The tones of the two books differ considerably. Saxe provides a calm and sus-

tained critique of the inadequacies of either Piagetian or Vygotskian theory for articulating how culture and cognition are related in the specificities of cultural practices. He devotes most of his book to a detailed exposition of a theoretical and research framework that treats children's development of facility with the cultural practices involved in candy-selling mathematics as a microcosm of the interdependence of culturecognition relations. Due to the broader ambitions of providing a theory of the social order that constitutes even the research practices of those who study culture-cognition relations, Lave devotes much of her book to a highspirited critique of the presuppositions that ground mainstream cognitive science and culture-cognition dichotomization, particularly assailing research on transfer of learning. Her enemies include "the colonization" of everyday cognition by education (p. 17), the narrow rationalistic psychology of problem solving, with its progression from lower to higher cognitive functions, and the "hegemony" of scientific thought (p. 82). One correlate of these views is the description of all research participants as "just plain folks," her replacement for the cognitive psychologist's ''subjects.''

### Lave's Cognition in Practice

This book documents a decade of research on "the occurrence, organization and results of arithmetic practice in everyday situations" (p. 47). The Adult Mathematics Project (AMP) studied 35 southern Californians through interviews and observations of their daily activities in grocery shopping, cooking, dieting, and money management.

There are three principle elements of the book: (a) descriptions of empirical results and a new theoretical framework for characterizing mathematical activities, (b) an epistemological critique of cognitive theories of problem solving, and (c) sketches of a theory of social order. These three elements meet with variable success. The first two are richly provocative, and the data and theoretical formulation Lave offers should contribute to widespread rethinking of what it means to do arithmetical activities. The third is not as well formulated in detail or implications.

# A New Framework for Mathematical Activity

Results from the AMP empirical studies are shown to be inadequately characterized by cognitive theory. One tour-deforce chapter compares differences in presuppositions, methods, findings, and conclusions between cognitive developmental studies of best-buy arithmetical thinking and Lave's in situ studies of grocery shoppers using mathematics, for whom new issues arose such as marginal value assessments, and managing food inventory and waste. Lave assembles a diversity of evidence for "situationally specific arithmetic practice," which she takes to be more representative of thinking than the general problem solving preeminent in cognitive theory. One striking characteristic of such situational arithmetic, for example, in calculating best buys, is that it is almost universally correct, whereas the same individuals score poorly when the same problems are presented in written form. Everyday arithmetic is also "actively constructed in series of transformations of relations of quantity" (p. 93); its shape and efficacy "depends upon its generation out of the articulation of structuring resources across occasions and situations" (p. 97).

"Dialectics" is the principle theoretical tool Lave uses to examine the constitutive influences of practice and structure. She argues that "cognition is constituted in dialectical relations among people acting, the contexts of their activity, and the activity itself" (p. 148). One of her central achievements is recasting problem solving from problems to solve to dilemmas to resolve-from a cognitive psychological perspective that tends to treat problems as givens, to a dialectical one that sees problem-solving activity in everyday situations as arising from conflict-generating dilemmas that require resolution:

The activity of finding something problematic subsumes a good deal of knowledge about what would constitute a (re)solution, or a method for arriving at one. . . . The dialectical process in the particular

context of everyday arithmetic is one of gap closing between the resolution characteristics and procedural possibilities. (p. 159)

Also fundamental is her concept of "structuring resources," familiar from microsociological theory and research by Garfinkel, Goffman, Sacks, and Schegloff, that is, those resources that come together "in a given situation to give quantitative relations their form and meaning" (p. 97), and may include memories of the person-acting, social relations, and the structuring of settings and their furnishings (including measurement devices, food shapes and sizes), in which activities are carried out. For example, units of calculation may be invented on the fly in ways shaped by the structuring resources available in the materials of the situation.

## An Epistemological Critique of Cognitive Theories of Problem Solving

Lave rightly harangues much of cognitive science as leaving absent from its analyses of problem-solving activities both the social situation of thinking and the motivation for problem solving and transfer of learning. In consequence of treating these factors as central in her framework for characterizing mathematical activities, she arrives at a radical view of knowledge as a process of dilemma-resolution rather than as factcommodity; she construes the "situation of activity" as dialectically constituted rather than as a given; and she redefines the typical means/end characterization of "problem solving" as a gap-closing process that unites means and ends (p. 175). Her correlated attacks on the epistemology of cognitive psychology and its conceptualizations of problem and mind/body separateness, among others, strike home on mainstream cognitive research. Nevertheless, other arguments may, to many readers, border on caricatures of the disciplines under critique, as when cognitive psychology is designated as "functionalist" in its core orientation to mind as a "self-perpetuating, closed, input-output system" (p. 191). For another example, not all learning research on transfer assumes, as she claims, that successful transfer will depend on "conscious attentive application of correct knowledge" (p. 63); cognitive research commonly documents knowledge transfer without such reflective awareness. There are also more varieties of cognitive science around than she acknowledges, some quite sympathetic to the dialectical framing of the emergence of problems and resolutions in activities (e.g., connectionists, neo-Vygotskians).

# Sketches of a Theory of Social Order

In the concluding chapter, the scope of the approach is broadened to that of the "analytic relations central to the dialectical problematic of constitutive order and the experienced lived-in world" (p. 178). The basic problem is that the constitutive order-the "contexts" which are part of her analysis of cultural practices (e.g., arithmetic in the supermarket as part of the sociopolitical economy)have their own contexts, as do the framing at this point in history of her own objects of empirical analysis, or those of "cognitive theory as a manifestation of Western culture" (p. 172). She thus sees the polar concepts of the culture-cognition paradigm as insufficient, requiring a theory of how the macrosocial order is constituted in order to theorize about the social production of action (including research, as well as supermarket shopping) in political economic structures (pp. 150, 177). The problems Lave points to in these formulations are real, but a critical discussion of the adequacy of her broader social theory will only become possible when the basic concepts of this system are worked out beyond the "capsule specification" (p. 178) she provides in the last 10 pages of the book, including an opaque social theory alternative to cognitive conceptions of transfer.

Lave's work will doubtless have influence on the conception of relations between mathematical practices and thinking. It would have been very useful if the arguments she develops were critically compared in their achievements with other social scientists. For example, Lave's key theoretical notion of context as continually cocreated by participants is reminiscent of Bateson's influential work on this concept in communication theory; this view is also shared in writings of discourse theorists, microsociologists, and ethnomethodologists. Curiously absent also are comparisons to Soviet psychologists such as Vygotsky and Luria, who seminally adopted the dialectical methods of Hegel and Marx for studying learning. In addition, Heidegger's treatment of the embodiment of thinking (recently made accessible by Dreyfus and by Winograd & Flores in their critiques of cognitive science and artificial intelligence) has a close similarity to many aspects of Lave's critique of mindbody dualism.

# Saxe's Culture and Cognitive Development

Saxe's investigations reveal the complex texture and learning resources of the cultural practices comprising candy selling by 5- to 16-year-old children in northeastern Brazil. His main achievement is the construction and exemplification of a constructivist research framework, summarized below, that targets a new level of analysis beyond the works of Piaget or Vygotsky-of activity in sociohistorical context—a level where culture and cognition are constitutive of one another, as Lave also argues. He is concerned with understanding the ways in which the shifting character of cognitive forms (e.g., currency arithmetic) for a learner are interwoven with shifts in the learner's goaldirected activities (e.g., converting wholesale candy prices to retail), in this case a practice organized mainly with respect to economic motives. In the practice, he found mathematical problems such as "the representation and comparison of large numerical values, additions and subtractions of large numerical values with currency, ratio comparisons of retail price units, and translations of wholesale box prices into retail unit prices with adjustments for inflation" (p. 66).

Saxe's framework has three analytic components: emerging goals, cognitive forms and functions, and culture-cognition interplay. Each component presents distinct methodological requirements and theoretical challenges for studying relations between sociohistorical processes and cognitive development, and brief examples of their content for his studies of candy sellers will convey how the framework structures the theory and research that results.

## Emerging Goals

In Saxe's detailed analyses of the enculturation process, specific mathematical goals emerge during participation in candy selling, and artifacts and conventions linked to practice give form to these goals. Studying goals requires in situ ethnographic analysis of learners of different levels of expertise in candy selling, and of goals that emerge through the learners' participations in

these cultural practices. These include four parameters of goals. The first is the general goal structure of the cultural activities, that is, selling candy, preparing to purchase wholesale, purchasing wholesale, and preparing to sell retail. The second is social interactions where goals are modified into cognitive forms (e.g., seller's territoriality can require evaluating competitors' prices; determining available cash for wholesale candy purchases calls for arithmetical computations with large currency values). The third parameter involves the use of cultural artifacts (currency of different

Saxe's main achievement is the construction and exemplification of a constructivist research framework, in which culture and cognition are constitutive of one another.

units, candy units per box), particular sign forms (number symbols), and conventions (price ratio pricing, such as 5 candies per CR\$1000). The fourth parameter is differences in the ways that understandings that sellers bring to the practice influence their emergent mathematical goals. To use Lave's language, these structuring resources give form to the situationally specific character of the goals that emerge, and thus to what Saxe calls the cognitive forms and functions that arise in accomplishing the practice-linked goals.

Sellers' prior understanding also interacted with emergent goals: Younger children were less agile with the mathematics (e.g., using a single pricing ratio, and simplifying arithmetical problems) but aroused more customer sympathy for sales and received more help in calculations involved in transactions.

#### Cognitive Forms and Functions

Saxe presented children with candyselling simulation tasks, standard number orthography tasks (e.g., to identify and compare numerical values), and arithmetical computation tasks. He wished to document the cognitive functions and forms that they used to accomplish the goals he found emerging in candy-selling practices. To study the development of cognitive forms and functions uniquely linked to the candy selling (e.g., to distinguish the effects of candy selling from use of the currency system), Saxe used these task performances to compare urban sellers to urban noncandy-selling agemates from the same economic community, and rural children who shared neither the practice nor the community.

Saxe finds that through participating in candy-selling practices, individuals construct specialized kinds of cognitive forms (e.g, currency arithmetic as a sign form for number representation) and functions (e.g., representing large numbers; setting candy prices; comparing price ratios) in order to accomplish their goals emerging through participation in candy-selling practices. Although children had limited knowledge of the standard number orthography, they used strategies involving an alternative representation system of currency-linked procedures rather than standard algorithmic forms for manipulating numbers on paper. Sellers did better at currency equivalence exchanges, correctly solved more complex computations, and used a distinctive strategy for ratio comparisons linked to selling practices. Saxe concludes that "first, with practice participation, sellers develop mathematical understandings distinct from non-practice participants; and second, these developments are ones that are interwoven with the mathematical and economic problems linked to the practice" (p. 99).

In another study using these tasks, Saxe illuminates how these differences developed by comparing sellers to nonsellers for three age groups: 5- to 7-year-olds, 8- to 11-year olds, and 12- to 15-year-olds. Among his findings are that complex mathematical goals such as ratio comparisons and wholesale-retail price translations did not arise for the youngest group.

## Culture-Cognition Interplay

To examine the interplay of culture and cognition, Saxe looks at transfer of learning of school-linked cognitive forms (algorithmic multiplication procedures) to address everyday practice-

linked problems (price translation) and vice versa. His developmental account views transfer "as a process of transforming prior cognitive forms into means of accomplishing new functions through a process of progressive specialization" (p. 174). He provides a fascinating analysis too complex to relate here on how selling practices lead to specialized cognitive forms and functions of price-ratio comparisons, and how school-linked procedures become specialized for component calculations of such comparisons.

Saxe's writing has a clarity and crispness of tone, and the analysis of data he provides in the context of illuminating the utility of the research framework is robust and well argued. Most importantly, one can imagine from the detailed example he offers how one might apply the theoretical and research framework to culture-cognition relations for other content domains.

Both of these books would be appropriate for advanced undergraduate and graduate seminars in psychology, anthropology, and education. The books are lush with excerpts from observation notes of learner activities and remarks. The two books, however, presuppose very different background reading acquaintance. Saxe, for the most part, provides reviews of the culture-cognition relations articulated in the works of Piaget, Vygotsky, and current scientists whose works are dependent on these frameworks. His important analysis of developments in cognitive form-function relations are indebted to Werner and Kaplan's Symbol Formation. Lave's book is fueled by and is more perceptible against the background of neo-Marxist social theorists such as Giddens, Bourdieu, and Sahlins, and to Frankfurt School critical theorists such as Adorno.

The core concept of "development" is treated very differently in the two books. "Development" is a normative concept, distinct from "change over time." It is fundamental for education, which is a prescriptive activity defining what ought to be. Saxe's perspective as developmental psychologist is to study the development of cognitive forms and functions of mathematics in relation to norms defined with respect to full participation in the cultural practices of candy-selling. For example, comparative analyses distinguish how persons structure their mathematical activities differently during candy selling as a function of years of participation in candy-selling practices. From all appearances, Lave's approach as anthropologist is to describe "what is" in situated mathematical activity without concern for developmental and comparative analysis. She considers all thought to have a "historically specific, circumscribed, and local character" (p. 193), and argues for the invidiousness of comparisons of lower/higher cognitive functions, or lower/higher levels of performance.

This stance is engendered only by Lave's social theory and by her study of adults, rather than by her perspective on mathematical activities. It leaves her work of questionable relevance for education because even for "just plain folks," some practice-linked behaviors are less acceptable, productive, or effective than others. Judging the state of the learner's activities against future desirable states has no place in her theory, which is without normative convictions. Of course she is right that comparisons can be used invidiously, but they can also be used to support developmental change.

Although arguments for particular norms of development are contestable, the fact that norms are used by communities for gauging proficiency of an individual's participation in a specific cultural practice, such as candy selling, is a social fact. The nondevelopmental stance of Cognition in Practice is also apparent in the glorification of the wellpracticed routines of "just plain folks," as if new learning and flexible adaptation to new structuring resources and problematics in situations were inconsequential. Yet this is just where a developmental analysis of the adaptiveness of cognitive form to function in what she would call "gap closing" is seen most saliently.

Within the constitutive order presented by a form of cultural practice, such as candy selling, one can surely articulate relatively well adapted forms and functions of mathematics for accomplishing the practice-linked goals constitutive of that practice. What her work neglects is that, given the same resources in a situation, different individuals—that is, those of different ages or, as in Saxe's studies, those with different levels of experience in cultural practices of candy selling—react in very different ways. People are not a single homogeneous group of "just plain"

folks." Nor is this group something to which individuals aspire, or which systems of cultural practices (or education) aim to promote—either in activities incidentally fostering learning or those which are arranged to do so.

Although the norms of development any community uses for gauging learning, progress, and so on, may be contested, how these norms function in the situated structuring of activities for different individuals in a cultural practice cannot be neglected. These issues are central to Saxe's studies of children and to education, but obviated by Lave's.

For many reasons, these books are likely to shake up mathematics educators and mathematics learning researchers. Saxe's and Lave's research frameworks and findings just do not fit within the adaptations of cognitive science views of problem solving that are predominant in mathematics education research. Even though that field calls for relevance of mathematics learned to everyday settings, there has been remarkably little ethnographic investigation of mathematical activities by children in settings outside classrooms. It is my belief that the critical insight uniting these two diverse books, and coming out of their methods of close, observational research in situ, is that problems emerge out of dilemmas, and that learning arises when means are sought to resolve these dilemmas. Sociocultural activities, involving artifacts, conventions, and human resources, provide the supportive environment that comes to structure how these dilemmas are resolved.

Finally and importantly, and also unlike most works in mathematics education, both books focus on learner competencies rather than failures in the activities that engage the participants. Saxe reveals the fascinating result that the mathematical goals that emerged for candy sellers differed in complexity as a function of age, but regardless of age, sellers were able to function effectively in practice. Lave finds that supermarket shoppers perform with remarkable accuracy the calculations they choose to perform in determining best buys. It is becoming clear to many researchers, including Lave and Saxe, that understanding how successful mathematical activities work will ultimately contribute more to advancing effective learning practices than repeated diagnoses of