An integrated perspective to approach technology in mathematics education

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AN INTEGRATED PERSPECTIVE TO APPROACH TECHNOLOGY IN MATHEMATICS EDUCATION

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Abstract

This paper concerns a research work developed in an European project. The aim of this work was to produce a document integrating the different theoretical frameworks employed by the project teams. The theoretical constructs of didactical functionalities, and experimental educational cycle, associated to an ICT tool, allowed us to analyse the roles played by technology in the considered set of theoretical frameworks. With this respect, we present examples concerning the theory of didactic situation, the activity theory and the theory of instruments of semiotic mediation.

Introduction

The project we are reporting on, is being developed in the framework of the Kaleidoscope Network of Excellence[1] which brings together European teams in technology-enhanced learning. Within the activities of the Network we are involved in the TELMA (Technology Enhanced Learning in Mathematics) project, which refers to the use of ICT (Information and Communication Technology) to improve mathematical education at school level.

The research teams involved in TELMA[2] aim at sharing their studies by discussing the following key themes: research area and goals, theoretical frameworks of reference, tools developed and/or used, contexts of use, work methodologies. A specific aim is to build, by means of a horizontal analysis, a document (IPTA) which represents an integrated in depth presentation of teams' approaches.

In this context, our specific work focuses on the theoretical frameworks of reference, and aims at integrating the different theoretical frameworks employed by the TELMA teams. Our working methodology is that of collecting and analysing ad hoc designed material: each team was required to write a presentation of its theoretical frameworks, and to present some selected papers. Because of the variety of the employed frameworks, an integrated vision was possible only through the definition of a perspective allowing us to analyse each framework pointing out common aspects and differences. In this paper we present such perspective giving examples of how it can be employed as a tool for analyzing different theoretical frameworks concerning the use of ICT in mathematics education.

1 Technology and mathematics educations

A first analysis of the collected material revealed that, the variety of theoretical

[1] "Kaleidoscope's goal is to integrate 76 research units from around Europe, covering a large range of expertise from technology to education, from academic to private research." (http://www-kaleidoscope.imag.fr/).

[2] Telma teams are the following: MeTAH and Leibniz – IMAG, Grenoble; DIDIREM University Paris 7 Denis Diderot; Istituto per le Tecnologie Didattiche (ITD) – C.N.R. of Genova; University of London (UNILON) - Institute of Education; Educational Tech Lab – NKUA University Athens.

frameworks depends on the involved ICT tools, and on the educational objectives addressed by each single research.

Two main kinds of ICT are involved in TELMA team's researches: those (e.g. Aplusix, l'Algebrista, ARI-LAB-2) which have been realized for explicit educational purposes (which we may call educational ICT), and those (e.g. CAS and Spreadsheet) that have been realized for professional purposes (professional ICT).

The researches involving educational ICT, in some cases, focus only on the use of ICT in educational practices, in other cases they consider the whole lifecycle of the tools, from the design to the actual use in educational practices and evaluation. In the case of professional ICT, TELMA teams have been focusing only on the educational use of the software, but not in their development.

Moreover it turns out that the teams address specific educational goals (for instance introducing pupils to symbolic manipulation, to geometry, to algebra, to proofs, etc.), referring to different theoretical frameworks and employing different ICT tools. In particular, we observed that a theoretical framework influences how a given ICT tool is employed in order to achieve a given educational goal, or in other cases it influences how an ICT tool is designed and developed to be used to achieve a given educational goal. This suggested us to consider the following primitives for our work: ICT tools, specific educational goals, how the ICT tools can be employed in order to achieve the given educational goals. We present a perspective, based on the concept of *didactical functionalities*, where we can define the relationships among such primitives.

2 Didactical functionalities of ICT tools

Given an ICT tool, and an educational goal, it is possible to identify its *didactical functionalities*:

> *With didactical functionalities we mean those properties (or characteristics) of a given ICT, and/or its (or their) modalities of employment, which may favor or enhance teaching/learning processes according to a specific educational goal.*

The three key elements of the definition of the *didactical functionalities* of an ICT tool are:

1. a set of *features/characteristics of the tool;*
2. a specific *educational goal;*
3. a set of *modalities of employing* the tool in a teaching/learning process referred to the chosen educational goal.

For what concerns the features and characteristics of ICT tools, we focus on the distinction between professional and educational ICT.

An educational ICT tool provides, because of its nature, a set of such functionalities. In fact we assume that the producers of the tool, not only design it with respect to a set of specific educational goals, but we assume that they also consider the possible modalities of employment of the tools in order to achieve such goals. In other words educational ICT tools are designed together with a set of *didactical functionalities*.

On the other hand professional ICT tools are not designed considering a possible educational goal and related modalities of employment: they are designed without a set of *didactical functionalities*. Nevertheless professional ICT tools may provide features that can be interpreted in terms of didactical functionalities, that is, we can identify modalities of employment of such tools aiming at the achievement of a given educational goal.

In general, the didactical functionalities can be defined/individuated either at the level of the design phase, or at the educational use phase. Thus in the case of professional ICT, the definition of didactical functionalities occurs only in the utilization phase, whilst in the case of educational ICT, they surely occur in the design phase, but may occur also in the educational use phase.

In the perspective we are proposing, in order to exploit a given ICT tool as a mean for achieving a given educational goal, it is needed to define the *modalities of employment* of the tool, which depend on the chosen theoretical framework of reference. In fact, in the researches of TELMA teams not only we found different theoretical frameworks, but we found also that ICT tools are employed in different phases of teaching/learning processes, and with different aims. For this reason we built a model allowing us to characterize such phases in which an ICT tool can be employed. The model is to be intended as a tool for classifying the modalities of employment defined by TELMA teams.

3 A model to classify the *modality of employment* of ICT tools

With respect to the definition of *didactical functionalities*, we shall observe that, given an ICT tool, the definition involves at least the tool itself, one learner and an interaction among them oriented toward a specific educational goal. However in the considered teaching/learning process other factors may play crucial roles. For instance, among the factors allowing an effective exploitation of the *didactical functionalities* of an ICT, we may consider: the context (is it on line, in class, or in a laboratory and so on), the proposed educational activities, the teacher and his/her strategies, national curricula etc.

TELMA teams employ ICT tools according to quite different modalities of employment. For this reason we developed a model, named *Educational Experiment Cycle* (EEC), to help us to classify such modalities (See Fig. 1).

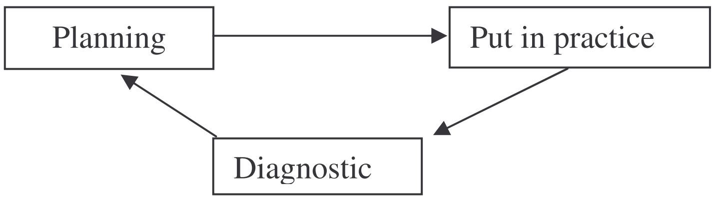

Fig. 1: Educational Experiment Cycle

The model attempts to describe the basic phases of a teaching/learning activity individuating three phases: the *planning* of the teaching/learning activity; the *put in practice* of the teaching/learning activity; the *diagnostic* phase.

Given an educational goal, the *planning* phase consists of the design and setting up of an activity (or sequence of activities) aiming at reaching the educational goal. The *put in practice* phase consists of the actual implementation of the planned activity (or sequence). The *diagnostic* phase consists of some evaluation of the actors involved in the cycle, could they be learners or teachers, with respect to the assumed educational goal.

4 Influence of theoretical frameworks on ICT tools didactical functionalities and on the Educational Experiment Cycle

In our perspective, the specific theoretical frameworks can be interpreted as instruments for defining the relationships among the primitives that characterise the concept of *didactical functionalities*. In this section we exemplify how the choice of a given theoretical framework can influence the definition of the didactical functionalities of ICT tools, either in terms of the design of the tools, or in terms of design of the modalities of employment. Moreover we show where the considered theoretical frameworks have been employed in different phases of the *EEC*.

The choice of the tools, and of the *modalities of employment* depend on the chosen framework of reference. Here we will consider (among the set of frameworks of TELMA teams) the theory of *didactic situations* (TDS) (Brousseau, 1986), the *Activity theory* (AT) (Nardi, 1996), and the theory of *instruments of semiotic mediation* (TISM) (Mariotti, 2002; Cerulli, 2004; Cerulli & Mariotti, 2003), and we will consider the example of symbolic manipulators employed to introduce pupils to symbolic manipulation. A comprehensive description of the three theories is beyond the scope of this paper, thus we limit ourselves to point out some key ideas and show how they can influence the definition of the didactical functionalities of ICT tools, and of symbolic manipulators in particular.

4.1 Defining didactical functionalities of an ICT according to the theory of didactic situations, in order to introduce pupils to symbolic manipulation

According to the TDS, learning happens by means of a continuous interaction between subject and milieu: each action of the subject in the milieu, is followed by a retro-action of the milieu itself, and learning happens through a spontaneous adaptation of the subject to the milieu, which is considered to be “milieu antagoniste” (Brousseau, 1986). One way of applying this key idea to the domain of educational ICT, is that of considering an ICT tool as an element of the milieu, and as such, its retroactions become a source for learning (by means of interaction with the ICT tool) in terms of the subject’s adaptation to the milieu.

Within this perspective, if we are given an ICT tool, a first *modality of employment* of the tool to achieve a given educational goal, is that of setting up a situation in which learners interact with the tool receiving a relevant feedback. In this case, the tool could be employed either during the *planning phase* or during the *put in practice* phase of the *EEC*.

For instance, suppose that a teacher wants to set up a situation, involving a symbolic manipulator, where the student is required to transform an algebraic expression into

another one, producing a chain of transformations. Following the TDS, the teacher may a-priori analyze the possible actions performed by the learner and the consequent retroactions of the symbolic manipulator. In this *planning phase*, of the *EEC*, the he/she may employ the ICT tool in order to investigate its retroactions. The teacher may thus individuate those retroactions that can be particularly relevant/effective for his/her specific educational goal, and, in the *put in practice phase,* he/she may submit to pupils a task that involves such particular retroactions.
For instance, if the focus is on the role of the brackets in algebraic expressions, and if the considered symbolic manipulator gives a particular feedback when the user tries to remove brackets from an expression, then the teacher may set up a task that involves removing of brackets in order to exploit the feedback provided by the software.
In summary, the TDS can be used in order to individuate *didactical functionalities* of an ICT tool with respect of a given educational goal, by defining its the *modalities of employment* in terms of setting up an ad hoc designed situation that exploit users' interaction with the ICT tool and the provided feedback.
If we want to design an educational ICT tool to be employed according to this perspective, special attention has to be paid to interaction issues and to the feedback offered. In the example that we discussed, the feedback could be very trivial or more complex; it could just inform the user if removing a couple of brackets is correct or incorrect, or it could also explain why the removal of a couple of brackets is correct or incorrect; it could allow incorrect removing of brackets signaling the error (or signaling nothing!), or it could simply not allow incorrect removing. Each of these different kinds of feedback could be exploited by setting up different kinds of situations. Actually among the researches of TELMA teams we find examples in which symbolic manipulators are developed within the perspective of the theory of didactic situations, and particular attention is paid to the feedback offered by the developed symbolic manipulators (Nicaud, 1994). In particular we find the example of Aplusix, developed within the framework of didactic situations by the IMAG team, which, in the case of incorrect removal of brackets, signals the error by means of a visual feedback (See Fig. 2).

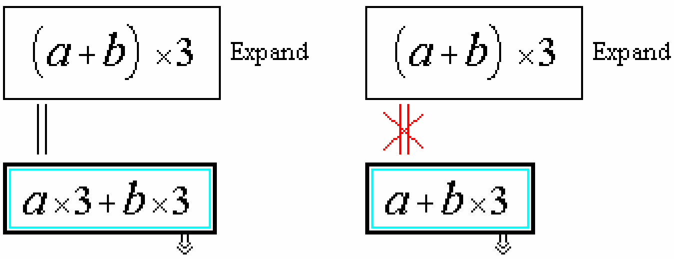

Fig. 2: Feedback in Aplusix in the case of incorrect removal of brackets a red cross appears between the old and the new expression.

4.2 Defining didactical functionalities of an ICT according to Activity Theory (AT), in order to introduce pupils to symbolic manipulation

The key concept of AT is the notion of *activity*, which is interpreted as a form of doing directed to an object. This theory provides a model to describe the structure of any human activity together with the transformations it undergoes during its evolution. The model, proposed by Engestrom and Cole (Nardi, 1996), concerns human activities in general, but can be used also to describe the system of relationships characterizing a teaching/learning activity. This model assigns a crucial mediation role to the instruments, the rules, and the division of labour in the three relationships characterizing any human activity, that is the relationships between subject and object, between subject and community, between community and object.

According to this theory an activity can evolve, during its development, when contradictions or breakdowns occur, forcing a change of focus in the activity, thus forcing a transformation of its structure. In other words, during the development of an activity, pupil's actions, teacher's actions, or other events can cause a change of the object or of the relationships characterising the activity itself; in this sense the teacher, which is a co-actor of the activity, can administrate/control/cause such changes, thus guiding the development of the activity according to his/her educational goal or to the exigencies of the class.

In this perspective, an ICT tool is not considered as antagonist to the subjects (as in the case of the *mileu antagoniste* of the TDS), on the contrary, it is considered to be a cooperative environment. When a learner uses an instrument for achieving an objective within an activity, the learning outcomes are considered to be structured by the nature of the activity itself and by roles played by all its components. Consequently, given an educational goal, the AT can be used to define the *modalities of employment* of a given ICT tool in terms of setting up an activity, involving the tool, and based on the cooperation of all participants. In other words, the *didactical functionalities* of the tool are defined in terms of how it can structure activities, rather than in terms of the retroactions given to the user as in the case of TDS; of course also such retroactions are to be considered, because they influence the relation between learner and tool, but they are not the main focus.

If we want to design an educational ICT tool to be employed according to the AT perspective, special attention has to paid to the tool's potentialities of interaction, communication and visualization.

Among the researches of TELMA teams we find examples in which symbolic manipulators are developed within the perspective of AT. Here we refer to the system of ARI-LAB-2, a software for the arithmetic problem solving, developed by the ITD (Istituto per le Tecnologie Didattiche - CNR Genova) team.

In this case the ICT tool is used by the team in the *planning* and *put in practice* phase. ARI-LAB-2 consists of a set of microworlds and two modes of interaction, the *teacher mode*, and the *pupil mode*. In the pupils mode it is possible to interact with the software solving tasks within one, or more, of the available microworlds. Not all the developed microworlds are always available to the pupils, in fact in the teacher mode it is possible to set up tasks to be submitted to pupils, and for each task it is possible to choose which specific microworlds shall be accessible to the pupil in order to solve the task. In other words the modalities of employment of the ITC tool involve both, the *planning* and the *put in practice phase*.

In particular, in the *planning phase* the ARI-LAB-2 can be used by the teacher to set up an activity (aimed at developing certain arithmetical competencies) in terms of defining the characteristics of the microworlds available to the user (See Fig. 3).
In the *put in practice phase*, learning is assumed to be an outcome of the planned activity which involves among other elements, the pupils and the ICT tool. As a consequence, configuring the tool, is a way, for the teacher, to define specific *didactical functionalities* as means for achieving her specific educational goals. In other words, the *didactical functionalities* are individuated in terms of the activities that can be set up and managed by the teacher.
For instance in order to introduce rules for adding fractions, the teacher can direct the focus back and forth between the fraction microworld, where the rules are explored dynamically and geometrically, and the symbolic manipulator microworld, where the rules are proven using a given set of axioms (See Fig. 3).

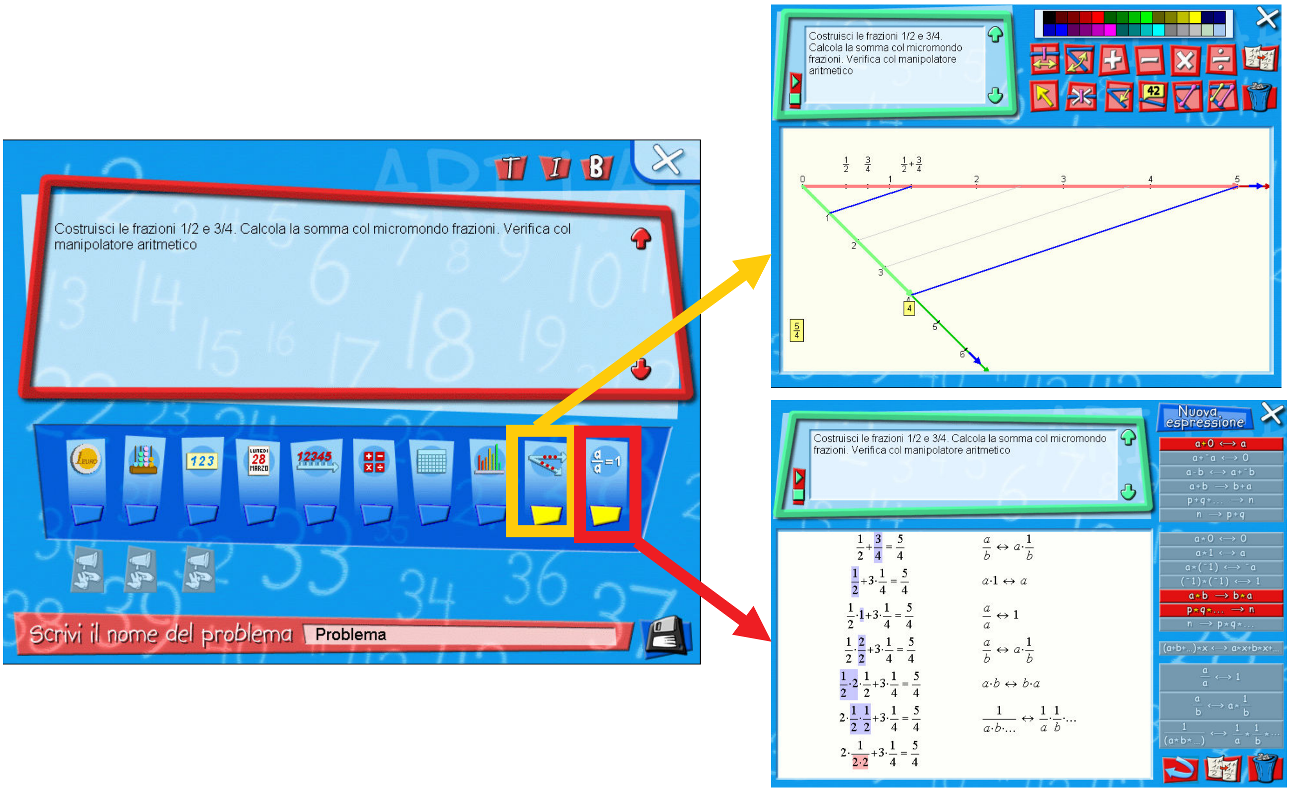

Fig. 3: In the teacher mode (on the left) the fraction microworld (top right) and the symbolic manipulator microworld (bottom left) are selected and are available for pupils' problem solving.

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4.3 Defining didactical functionalities of an ICT according to the theory of instruments of semiotic mediation, in order to introduce pupils to symbolic manipulation

A different perspective is that of the theory of the *instruments of semiotic mediation* (TISM), which, like the AT, is derived from the theories of Vigotskij. The key hypothesis of this theory is that meanings are rooted in the phenomenological experience, but they can evolve, under the guidance of the teacher, by means of special communication strategies (Mariotti, 2002), such as for instance that of the mathematical class discussions (Bartolini Bussi, 1996). Without going deeply in detail in the description of this theory, we observe that it assumes that a part of the teaching/learning process happens at the semiotic level, and that it depends strictly on the signs that can be derived from the considered ICT tool, and can be employed by the teacher as means for orchestrating relevant mathematical class discussions. In other words the *modalities of employing* an ICT tool within this perspective consist on the one hand of setting up ad hoc designed activities involving the tool, and on the other hand of orchestrating mathematical discussions using signs derived from the ICT tool. Consequently for this theory, it is particularly important to study what kinds of signs (words, formulas, gestures, etc.) can be derived from a given ICT tool in order to orchestrate a mathematical discussion relevant for the chosen educational goal. For instance, if we take the example we discussed previously in the case of TDS, we considered the issue of removing brackets in algebraic expressions. Such an issue has been addressed by the ITD team of TELMA when they developed the symbolic manipulator L'Algebrista (Cerulli 2004), and the chosen strategy was that of providing the software with a button to be used to remove brackets; such a button does not check if the operation is correct or not, it just executes it, thus it may produce incorrect transformations of algebraic expressions, giving no feedback. However the interface of the software associates a formula to the button ("**(a+b) → a+b**"), together with a peculiar name "risky button" which is used by the teacher in the *put in practice phase*, during mathematical discussions, as a means for focusing pupils attention on the "risks" of removing brackets from algebraic expressions. In this case the provided feedback is not the most important element contributing to the achievement of the educational goal. In fact the most important element is the communication strategy that can be developed by the teacher with reference to the ICT tool.

4.4 Employing ICT tools in the *diagnostic phase* of the *EEC*

ICT tools can be employed for educational purposes at any stage of an *EEC*, exploiting their *educational functionalities* as means for reaching a given educational goal. The examples we presented concern the *planning* and the *put in practice phase* of the *EEC*, however, among TELMA researches we find also examples concerning the *diagnostic phase*.

For instance in the Lingot project (http://pepite.univ-lemans.fr/), the DIDIREM team research aims at developing diagnostic and remedial tools in elementary algebra, testing them with students and also studying how teachers appropriate the use of such tools. The hypothesis is that there exists some coherence in student's behavior. Thus understanding this coherence and how it can evolve is a necessity for developing effective diagnostic and didactic strategies based on this diagnostic. Then, the TDS is used for supporting the conception of tasks linked to the diagnostic. In this case, the definition of the *modalities of employment*, of the used ICT tool, is based on the idea that the teacher submits to pupils a *diagnostic* activity based on the tool, and the feedback received by the teacher is used as a basis for *planning* (according to the TDS) the tasks to be submitted to pupils in the *put in practice* activity. In other words the ICT tools is employed in the diagnostic phase of the *EEC*, and the provided feedback contributes to the setting up (*planning phase*) of the situations to be submitted to pupils, in order to achieve the given educational goal in the *put in practice* phase. The peculiarity of this perspective is that the ICT tools are employed by the teacher as sources of information rather then as mediators directly fostering pupils learning: the didactic (or adidactic) situations, planned for fostering learning may even not include an ICT tool at all, even if they have been planned with the aid of a diagnostic ICT tool.

4.5 Some remarks and conclusions

We observe that the designer of an educational ICT tool, provides it with a certain set of *didactical functionalities* according to a given theoretical framework of reference. However it may happen that someone else decides to employ the same tool to achieve the same educational goal, but taking the perspective of another framework of reference. If that is the case, the individuated *didactical functionalities* will be different from those implemented by the designer. An example is the research brought forward by the University of Siena team where the Cabri-Geometry software for introducing pupils to geometrical constructions, designed within the framework of TDS, is used by the team according to the TISM (Mariotti 2002). In this case the *didactical functionalities* defined by the developer of the software are different from the *didactical functionalities* defined by the TELMA team because even if educational goal and ICT tool coincide, the *modalities of employment* are different.

It is interesting to observe that in this example, like in the other examples we presented, ICT tools are provided with very different didactical functionalities, depending on the different theoretical frameworks that assign very different roles to the tool itself, to the learners, and to the teacher.

We considered the case of, TDS that is based on Piaget's theories, according to which, the development creates the condition for the process of learning. Nevertheless the development follows stages that are universal and common to any individual of any culture. In this context, in order to teach a mathematical concept, it is important that the teacher, in the *planning phase*, sets up a fundamental situation (adidactic situation) which will be the point of departure to create an antagonist system for pupils, the milieu, which includes the ICT tool. The role of the teacher is to construct the condition under which the responsibility of the solution of the task is entirely submitted to the student in the *put in practice phase*; in this phase the interaction between student and tool (included in the milieu) is the main source of learning.

On the other hand AT and the TISM, are both based Vigotskij's socio-historical theory. In this theory the student's cognitive development has to be understood as taking place in the interaction with other members of the society, in particular with the teacher and other members of the class. In this perspective, the teacher assumes a key role in the *put in practice phase*, for instance in the TISM, the teacher plays the central role of orchestrating mathematical discussions arising from students interaction with the ICT tool.

In all these cases, the learning outcomes depend strongly on the tools used, but in the case of Vigotskijan theories we find a strong dependence on cultural settings which may not be so relevant in the case of Piagetian theories such as TDS. A direct implication is that when defining the didactical functionalities of a tool, a different attention is put on the social context according to the theoretical framework used.

In conclusion, we showed how the constructs of didactical functionalities and the *EEC*, allowed us to analyse the roles played by technology in some examples. We hypothesize that the introduced constructs can be used to extend the analysis and comparison of researches, including also researches outside the TELMA project but concerning educational use of ICT tools.

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