The Impact of Problem Order: Sequencing Problems as a Strategy for Improving One’s Performance

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Abstract
Two experiments investigated the impact of problem order and problem sequencing on performance. In experiment 1 subjects were either presented with a suitable or an unsuitable presentation sequence where they were free to deviate from. Presentation sequence had an impact on performance and rearranging problems improved performance for high prior-knowledge subjects whereas low prior-knowledge subjects’ performance deteriorated. Experiment 2 yielded evidence that effects of problem sequence have to be triggered by directing subjects’ attention to comparing problems before working on them. Results are discussed within the framework of analogical transfer.

The Impact of Problem Order
In this paper we investigate the impact of problem order on performance when solving a sequence of mathematical problems. It has to be noted that effects of sequencing to-be-learned materials have been widely studied in the Sixties and the Seventies (Posner & Strike, 1976; Van Patten, Chao, & Reigeluth, 1986 for an overview), whereas effects of sequencing to-be-solved problems have received only little attention. Sequence effects are said to occur when performance on problem B varies depending on whether problem A had been performed before or not. This influence of solving problem A on performance for problem B should be specific to problem A, i.e., solving a problem C before B should not necessarily lead to the same performance for B as solving problems in the sequence AB. This specificity assumption distinguishes sequence effects from mere training or position effects.

Sequence effects can be analyzed as the result of two distinct cognitive processes that take place in succession, namely, learning and transfer. Learning refers to a change in the cognitive system of the problem solver (i.e., newly generated or modified knowledge structures) that occurs due to solving a problem A. Transfer refers to the transmission of these newly generated or modified knowledge structures to a subsequent problem B.

The two most prominent approaches to transfer are Singley and Anderson’s rather analytical theory on transfer of cognitive skill (Singley & Anderson, 1989) and the more holistic theories of transfer by analogy (Gentner, 1983; Gick & Holyoak, 1980).

Singley and Anderson’s basic assumption is that a problem is more likely to be solved the more declarative and/or procedural knowledge elements necessary to solve that problem are already known by the problem solver. Therefore, transfer among problems should increase with the number of elements being shared by the problems (Thorndike & Woodworth, 1901). Furthermore, because transfer is based on the extent of overlap between the knowledge structures necessary to accomplish two tasks a symmetrical relation between problem A and B is assumed. (Pirolli & Recker, 1994; Singley & Anderson, 1989). It is important to note, however, that sequence effects may be asymmetrically, i.e., a problem sequence AB might result in a different performance than a problem sequence BA. This asymmetry is due to the fact that the amount of what has been learned in the first place and can therefore be transferred to a succeeding problem may differ among problems. For instance, working on a difficult problem at first may result in less learning than starting with a simpler problem.

Transfer by analogy is described as the transmission of knowledge from one problem-solving situation (the source) to a target problem and consists in a number of different processes. In order to solve a target problem first a suitable source problem has to be retrieved from memory. Next, elements of the source problem have to be mapped onto the target problem. Finally, based on these mappings a solution for the target problem is generated. Research on analogy has demonstrated that structural similarity among source and target is the most important determinant of successful transfer and that this transfer is often restricted to situations where source and target are structurally equivalent. If there are structural differences between problems subjects often fail to adapt a source problem’s solution to fit the requirements of the target (Reed, Dempster, & Ettinger, 1985).

With regard to sequence effects it can therefore be assumed that performance for a specific problem should improve if one solves structurally similar problems in succession. Contrarily, switching between unrelated problems might impede problem solving because this increases the probability that unsuitable preceding problems are used as sources to guide later problem solving.

Whereas there is only preliminary evidence for this assumption concerning structural similarity in a study by Novick (1988), two problem-solving studies have
investigated the effects of the second aforementioned factor that may influence the suitability of a problem sequence, namely a problem’s difficulty. Reed, Ernest, and Banerji (1974) obtained no effect of problem order when studying transfer from the easier Missionary-Cannibals problem to the more difficult Jealous-Husbands problem and vice versa. Subjects who had been acquainted with the similarity relations among the problems, however, solved the problems faster in the difficult-easy sequence. Furthermore, Cook (1937) found that a difficult-easy sequence led to better performance when working on pyramid puzzles. Based on these two results it could be argued that solving difficult problems before easier ones should result in better performance than a reversed sequence. However, this may only hold for knowledge-lean problems (in the sense of VanLehn, 1989) whereas for knowledge-rich problems solving an easy problem first may support solving more difficult problems of the same problem category. This should be the case because more difficult problems often share structural elements with the easier problems. Therefore, mastering these problem components in the easier problems provides practice for solving the more difficult problems entailing these components among other new elements. This idea of mastering (subordinate) parts of a skill before proceeding to more difficult demands is in line with proposals made for the design of instructional curricula (cf. Schoenfeld, 1985; Van Patten et al., 1986).

To summarize, problem sequences that are ordered with respect to the structural similarity of the problems (similar problems being solved in succession) and with respect to the difficulty of the problems (easy-to-difficult) should result in better performance compared to either reversed or to random sequences.

**Sequencing as a Metacognitive Strategy**

In experimental problem-solving settings subjects are usually asked to maintain a given order when solving multiple problems whereas in more self-controlled situations they might be given the opportunity to decide on a problem sequence by themselves. In this case the question whether problem solvers strategically rearrange problems in order to improve their performance gains increasing importance.

Problem sequencing can be seen as a process that is exactly reverted to the retrieval process in analogical problem solving. In analogical problem solving a backward search is conducted to find a source problem in memory whose solution can be adapted to the to-be-solved target. Contrarily, sequencing may be described as a forward search to decide on the next to-be-solved problem (target) for which the solution of the problem being solved most recently (source) can be adapted. Conceptualizing problem sequencing in accordance with the retrieval process in analogical problem solving brings about some major advantages. In particular, findings on analogy may be used to derive hypotheses concerning problem sequencing as a metacognitive problem-solving strategy.

First, the propensity to sequence problems should depend on whether subjects are aware of the fact that different problem sequences may be associated with different performance outcomes and that applying knowledge used to solve one problem when approaching a next problem might foster performance. However, research in analogical problem solving has repeatedly shown that subjects often fail in using previous problem-solving experiences spontaneously when solving new problems (Reed et al., 1985) and that they need to be provided with hints in order to ensure analogical transfer (Gick & Holyoak, 1980). Additionally, the costs that result from searching for a suitable target problem that is to be solved next have to be less than the benefits that are achieved by rearranging problems deliberatively. This reasoning is in line with assumptions made by Novick (1988) or Reed et al. (1974) concerning the retrieval process in analogical problem solving. For instance, Reed et al. (1974, p. 448) postulate that “the total time to retrieve, translate, and use analogous information to find an operator should be less than the total time to find the same operator without using information from the previous problem.”

Second, the successfulness of rearranging problems depends on whether subjects are able to identify a suitable problem sequence by themselves. In terms of analogical problem solving this relates to the question of whether subjects are able to retrieve a source problem that is structurally similar to the target. With respect to this issue research has demonstrated that subjects often face difficulties in recognizing structural problem features and that they are often misled by surface similarities of the problems (Holyoak & Koh, 1987; Ross, 1987). Novick (1988) demonstrated that the ability to retrieve a structurally similar analogue interacts with subjects’ domain-specific prior knowledge with experts being more likely to find a suitable source problem than novices are. Therefore, it can likewise be assumed that the quality of problem sequencing might interact with subjects’ prior knowledge in a way that only high prior-knowledge subjects benefit from self-determined sequences whereas the additional freedom of rearranging test problems might even be harmful for less advanced subjects.

In order to investigate the impact of problem order and problem sequencing on problem-solving performance two experiments were conducted. In experiment 1 subjects were provided with one of two different presentation sequences that they were free to rearrange. Contrarily, subjects in experiment 2 were confronted with predefined problem orders they could not deviate from in order to find out whether differences in problem-solving performance can still be observed when subjects are not made aware of the potential impact of problem order.

**Experiment 1**

**Method**

**Participants** Subjects were 76 students (49 female, 27 male) of the University of Goettingen, Germany, who participated for course credit or payment. Average age was 22.67 years.
Materials and procedure For experimentation the hypertext-based learning and problem-solving environment HYPERCOMB was used (Gerjets, Scheiter, & Tack, 2000) which contains a short introduction to the domain of combinatorics followed by a learning phase where subjects can acquire knowledge by studying worked-out examples for six problem types.

Permutation problems are about finding out the number of possibilities of bringing all elements of a set into a distinctly ordered arrangement. Variation problems deal with the number of possibilities for selecting a subset of elements out of a set of elements in a distinct order. Combination problems are about the number of possibilities for selecting a subset of elements out of a set of elements without regard to the order. All three kinds of problems can be further distinguished as being with or without replacement yielding six problem types. Replacement indicates whether the set contains undistinguishable elements or whether elements can be selected more than once, respectively. Similarity among permutations, variations, and combinations can be described with respect to the number of permutations necessary to solve a specific problem. These similarity relations among the problem types are not only expressed at this conceptual level but are also reflected at the computational level in the graded complexity of the formulas needed to solve the problems. Therefore, one can characterize transfer relations among problem types in combinatorics that are based on the overlap at the computational and conceptual level. According to this task analysis a problem sequence ranging from permutations to variations and ending with combinations should be suited best for problem solving as the problem types that are most structurally similar to each other are presented in succession.

In HYPERCOMB each problem type was illustrated by abstract information concerning its structural features and two worked-out examples. One example explained the basic application of the solution principle and the other example illustrated a more complicated situation where the solution principle in question had to be applied twice in order to solve a problem. Subjects could decide which instructional materials they wanted to study and when they wanted to quit the learning phase. In the subsequent test phase the instructional material was no longer available and subjects were asked to work on six test problems. For those test problems the solution principles which had been taught before had to be applied once for easy problems or twice for difficult problems (figure 1).

When starting the test phase subjects were informed that they would have to solve six test problems listed on a single page. They were asked to study all test problems carefully before selecting a problem they wanted to start working on. Subjects were further informed that they could solve the test problems in any order they wanted. Whenever subjects had solved a problem the initial page with all six problems was presented (including the ones already being solved) and subjects were asked to select the next problem. In order to prevent subjects from solving a problem twice solved problems could no longer be retrieved.

Easy problem: A lighthouse can flash in six different colors (red, yellow, green, blue, orange, pink) from which colors are randomly chosen to form a flare. Each flare contains two colors in succession and none of the colors can appear twice in one flare. What is the probability that the lighthouse will send a red-orange flare, i.e. it will first flash red and then flash orange?

Difficult problem: At a soccer game there are two dressing rooms for the two teams. The kickers from Oxford wear T-shirts with uneven numbers from 1 to 21 and Manchester has even numbers from 2 to 22. As the aisle from the dressing rooms is very narrow only one player at a time can enter the field. The players of the two teams leave their rooms alternately with a player from Oxford going at first. What is the probability that the first five players who enter the field have the numbers five, two, thirteen, eight, and one (i.e., the first has the number five, the second has got the two and so on)?

Results and Discussion A first comparison by means of an ANOVA (presentation sequence x prior knowledge) revealed no significant differences with regard to either pretest errors ($F(1,72) = 1.29; MSE = 83.10; p > .25$) or overall example-processing time ($F < 1$) between the presentation sequences (table 1).

In order to analyze subjects' performance on the six test problems and on subjects' problem-solving time as a function of presentation sequence, prior knowledge, and sequencing behavior a third factor was entailed in the
analysis. This factor indicated whether subjects had kept the presentation order while working on the problems or whether they had deviated from it (i.e., sequencing behavior). Additionally, we used example-processing time as a covariate because this turned out to be a very important factor for predicting subjects’ performance and because this measure was characterized by a high variability within each of the two presentation sequence conditions. This resulted in a three-factor ANCOVA (presentation sequence x prior knowledge x sequencing behavior) that was deployed for analyzing performance on easy and difficult problems as well as for problem-solving time. We will first report the effects for presentation sequence and prior knowledge (table 1) before having a closer look to the impact of subjects’ sequencing behavior on performance (figures 2a, 2b).

Table 1: Performance (in %) and time data (in sec) as a function of presentation sequence and prior knowledge

<table>
<thead>
<tr>
<th>Presentation sequence</th>
<th>Suitable sequence</th>
<th>Unsuitable sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior knowledge</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Pretest errors</td>
<td>44.3</td>
<td>74.1</td>
</tr>
<tr>
<td>Example-processing time</td>
<td>650</td>
<td>547</td>
</tr>
</tbody>
</table>

Effects of sequencing behavior
A question that has yet been left unanswered is whether subjects rearrange problems when being confronted with an unsuitable presentation sequence and how their sequencing behavior contributes to problemsolving performance. Analyzing the percentage of subjects who deviated from the presentation sequence an ANOVA (presentation sequence x prior knowledge) clearly revealed that subjects reacted sensitively to the quality of the presentation sequence by deviating more often from the unsuitable sequence than from the suitable one ($F(1,72) = 1.78$ and all $F$s < 1). Deviations from the given presentation sequence were mainly caused by subjects’ preference to work on easy problems before approaching the more difficult ones - regardless of structural similarities among easy and difficult problems.

Effects of sequencing
With regard to the impact of sequencing on performance for easy problems an expected pattern of results could be obtained (figure 2a). There was no main effect of sequencing behavior ($F < 1$), however, sequencing behavior interacted with subjects’ prior knowledge in that high prior-knowledge subjects improved by rearranging problems whereas low prior-knowledge subjects’ performance even deteriorated ($F(1,72) = 5.20; MSE = 244.29; p < .05$). Although this effect seemed to interact with presentation sequence the triple interaction was not significant ($F(1,72) = 1.31; MSE = 244.29; p > .20$), nor was there an interaction between presentation sequence and sequencing behavior ($F < 1$).

The effects for problem-solving performance on difficult test problems was different (figure 2b). Performance improved slightly by rearranging problems ($F(1,72) = 2.99; MSE = 371.93; p < .10$), whereas there were no interactions with prior knowledge or presentation sequence (all $F$s < 1). Additionally, there was no main effect for sequencing behavior on the overall time subjects needed to solve all six test problems ($F < 1$). Although there were no interactions with either presentation sequence or prior knowledge (all $F$s < 1.78 and all $p$s > .15).
To summarize, we found that the order in which problems are solved had an impact on problem-solving performance for easy test problems in that a sequence where problems were arranged according to their structural similarity and difficulty was superior to a presentation sequence not making use of this principle. Additionally, we could demonstrate that subjects tried to make use of this effect of problem sequence by rearranging problems when they were presented in an unsuitable way. However, improvements due to problem sequencing were predominant for subjects with high prior knowledge who are more likely to identify structural similarities among problems. On the contrary, low prior-knowledge subjects’ performance deteriorated when they deviated from a given order of problems if these problems were unsuitably arranged.

However, the pattern of results obtained for difficult test problems yielded evidence for some additional speculations. In particular, sequencing improved performance on difficult test problems independently of whether subjects deviated from a suitable or an unsuitable sequence. Subjects who rearranged problems may have followed the instruction to first read all problems carefully before selecting a problem to work on. This may have focussed subjects’ attention on comparing test problems and thereby displaying a deeper processing which in turn improved performance. This interpretation is related to the question on whether subjects spontaneously notice problem similarities by themselves or whether they need hints in order to make use of potential analogues relations among test problems. If the latter is true, sequence effects should only be observable when subjects are asked to process test problems thoroughly as in this experiment, but should be absent when problems are presented in predefined orders without any further instructional support. In order to address this issue a second experiment was conducted.

Experiment 2

Method

Participants Subjects were 78 students (48 female, 30 male) of the University of Goettingen who participated for course credit or payment. Average age was 24.1 years.

Materials and procedure The same learning and problem-solving material as in experiment 1 was used. However, the procedure was varied. The problems were presented in predefined sequences that subjects could not deviate from. Subjects started working on problem 1 in the sequence. After subjects had solved a problem the next problem was automatically presented. Subjects did not see any of the test problems before this automatic presentation. No return to preceding problems was possible.

Design and dependent measures As a first between-subjects variable the presentation sequence was varied by presenting the problems according to the same orders as in experiment 1. As a second between-subjects variable subjects’ domain-specific prior knowledge was used. As performance measures subjects’ error rates and problem-solving time were registered. Additionally, example-processing time was measured.

Results and Discussion

A first comparison by means of an ANOVA (presentation sequence x prior knowledge) revealed no significant differences with regard to prior knowledge between the two presentation sequences ($F < 1$). The effect of presentation sequence for example-processing time however almost reached statistical significance ($F(1,74) = 2.49; MSE = 1555590.10; p > .10$). Therefore, this variable was again used as a covariate in all further analyses (table 2).

Table 2: Performance (in %) and time data (in sec) as a function of presentation sequence and prior knowledge

<table>
<thead>
<tr>
<th>Prior knowledge</th>
<th>Suitable sequence</th>
<th>Unsuitable sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest errors</td>
<td>High: 46.8, Low: 74.7</td>
<td>High: 47.2, Low: 70.8</td>
</tr>
<tr>
<td>Example-processing time</td>
<td>722, 678</td>
<td>539, 579</td>
</tr>
</tbody>
</table>

**Problem-solving errors:**

- Easy problems: 16.1, 21.4, 15.8, 18.1
- Difficult problems: 35.9, 49.3, 42.4, 50.8

**Problem-solving time:** 977, 897, 830, 849

There was no main effect for prior knowledge on performance for easy test problems ($F(1,74) = 1.04; MSE = 258.55; p > .30$), whereas it positively influenced performance on difficult test problems ($F(1,74) = 7.20; MSE = 321.39; p < .01$). Most interestingly, there were no effects
of presentation sequence and no interactions between the two factors for any of the two performance measures (all $F$s $< 1$). Additionally, presentation sequence had barely no impact on problem-solving time ($F(1,74) = 1.89$; $MSE = 46501.88$; $p > .10$). The main effect for prior knowledge as well as the interaction were not significant ($F$s $< 1$). The interpretation of these results is straightforward. Simply presenting test problems in a suitable order is obviously not sufficient to improve problem-solving performance.

**General Discussion**

In experiment 1 a problem sequence where problems were arranged according to their structural similarity and their difficulty outperformed a problem sequence where these sequencing principles were reversed. Experiment 2 demonstrated that sequence effects only occurred when subjects were instructed to process problems carefully before working on them. This is in accordance with findings on analogy that spontaneous transfer is hard to achieve. Instead, subjects need hints that relations between problems are important in order to benefit from a suitable sequence.

Additionally, we demonstrated that subjects try to make use of this effect of problem sequence by rearranging unsuitable problem sequences. However, only subjects with high prior knowledge who are more likely to identify structural similarities seem to benefit from problem sequencing. In contrast to that, subjects with low prior knowledge do not seem to possess the skills necessary for identifying a more suitable problem sequence than the one they are initially presented with.

Several issues will be addressed in forthcoming experiments. First, the question arises whether subjects’ ability to sequence problems as well as spontaneous transfer within predefined problem sequences can be fostered by deliberately directing subjects’ attention to structural similarities of the problems. Second, it is of interest whether other findings of analogy-based research can likewise be transferred to problem sequencing. In particular, we want to investigate whether not only the retrieval process in analogical problem solving but also problem sequencing is vulnerable to effects of superficial similarities among problems. Third, we aim at distinguishing sequence effects that occur due to structural similarity versus sequence effects that are merely caused by the relative difficulty of problems. Additionally, a more-fine grained analysis of subjects’ sequencing strategies with regard to this distinction seems promising. The results of experiment 1 provide preliminary evidence that subjects mainly sequenced problems according to their relative difficulty without paying attention to their structural interrelationships. In domains where structural similarities among problems are more evident - like algebra word problems - sequencing behavior may be quite different. Therefore, a series of experiments is currently being conducted using algebra problems.

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**References**


