# Report on visual interactive building blocks identified in current national mathematics curricula 

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## To cite this version:

Marta Turcsanyi-Szabo. Report on visual interactive building blocks identified in current national mathematics curricula. 2004. hal-00190148

HAL Id: hal-00190148
https://telearn.hal.science/hal-00190148
Submitted on 23 Nov 2007

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Concepts and methods for exploring the

future of learning with digital technologies

# D27.1.1 (Final) Report on visual interactive building blocks identified in current national mathematics curricula 

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Document classification : PU
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# KALEIDOSCOPE JERP Building Visual Interactive Blocks for Tangible Mathematics 

Trail 1 Report<br>WP. Leader: Márta Turcsányi-Szabó

2004-06-04

## TABLE OF CONTENT

Main objectives ..... 3
Phase1: Framework for identifying levels of attainment with numbers ..... 3
Phase2: Application of framework to collect data ..... 6
Phase3: Analysis of items for common components of international relevance ..... 8
3D shapes ..... 9
Animations ..... 11
Bigger or smaller ..... 12
$\square$ Charts, pie-charts and graphs ..... 14
$\square$ Coordinates, coordinate-systems ..... 16
$\square$ Counting ..... 19
$\square$ Fractions. ..... 21
$\square$ Games with numbers ..... 28
$\square$ Geometry, angles ..... 33
Numbers in life ..... 34
Logo ..... 38
Money ..... 39Music and math's44
Number lines ..... 45
$\square$ Operating with numbers ..... 47
$\square$ Pin board ..... 51
$\square$ Probability ..... 53
$\square$ Sequences ..... 54
$\square$ Shapes ..... 57
$\square$ Symmetry ..... 59
$\square$ Temperature ..... 62
$\square$ Time ..... 63
$\square$ Words for math's ..... 65

## Main objectives

## The main objective of this JEIRP is to:

- document and analyse partners' national mathematics curricula for children aged 8 to 12 with the goal to identify and select several elementary building blocks (tools, representations, gadgets or components),
- prototype them as interactive, tangible and visual programmable objects,
- evaluate how these building blocks could be used to author computer environments in which children would learn about mathematics concepts and relations in a highly interactive and visual way, and
- explore feasibility of this approach, the way how it could be scaled up and how educators in different countries could further develop its potential.


## This report covers the finding of Trail 1

## Trail 1 - Identifying examples of visual interactive building blocks in various national mathematics curricula for children aged 8 to 12 (WP leader: Marta Turcsanyi-Szabo)

We specify an agreed framework for the procedure by which candidate examples from each National Curriculum are identified. In the second phase, that framework is applied. The third phase will be one of negotiation in which the candidate items will be analyzed for common components of international relevance, resulting in deliverable D1 - Report on visual interactive building blocks identified in current national mathematics curricula.

## Phase1: Framework for identifying levels of attainment with numbers

Here we found that most of the attainment targets are quite similar to that found in the UK description, thus we built our identification framework accordingly:

## Level 1

Pupils count, order, add and subtract numbers when solving problems involving up to 10 objects. They read and write the numbers involved.

## Level 2

Pupils count sets of objects reliably, and use mental recall of addition and subtraction facts to 10 . They begin to understand the place value of each digit in a number and use this to order numbers up to 100. They choose the appropriate operation when solving addition and subtraction problems. They use the knowledge that subtraction
is the inverse of addition. They use mental calculation strategies to solve number problems involving money and measures. They recognise sequences of numbers, including odd and even numbers.

## Level 3

Pupils show understanding of place value in numbers up to 1000 and use this to make approximations. They begin to use decimal notation and to recognise negative numbers, in contexts such as money and temperature. Pupils use mental recall of addition and subtraction facts to 20 in solving problems involving larger numbers. They add and subtract numbers with two digits mentally and numbers with three digits using written methods. They use mental recall of the $2,3,4,5$ and 10 multiplication tables and derive the associated division facts. They solve wholenumber problems involving multiplication or division, including those that give rise to remainders. They use simple fractions that are several parts of a whole and recognise when two simple fractions are equivalent.

## Level 4

Pupils use their understanding of place value to multiply and divide whole numbers by 10 or 100 . In solving number problems, pupils use a range of mental methods of computation with the four operations, including mental recall of multiplication facts up to 1010 and quick derivation of corresponding division facts. They use efficient written methods of addition and subtraction and of short multiplication and division. They add and subtract decimals to two places and order decimals to three places. In solving problems with or without a calculator, pupils check the reasonableness of their results by reference to their knowledge of the context or to the size of the numbers. They recognise approximate proportions of a whole and use simple fractions and percentages to describe these. Pupils recognise and describe number patterns, and relationships including multiple, factor and square. They begin to use simple formulae expressed in words. Pupils use and interpret coordinates in the first quadrant.

## Level 5

Pupils use their understanding of place value to multiply and divide whole numbers and decimals by 10, 100 and 1000. They order, add and subtract negative numbers in context. They use all four operations with decimals to two places. They reduce a fraction to its simplest form by cancelling common factors and solve simple problems involving ratio and direct proportion. They calculate fractional or percentage parts of quantities and measurements, using a calculator where appropriate. Pupils understand and use an appropriate noncalculator method for solving problems that involve multiplying and dividing any three-digit number by any two-digit number. They check their solutions by applying inverse operations or estimating using approximations. They construct, express in symbolic form, and use simple formulae involving one or two operations. They use brackets appropriately. Pupils use and interpret coordinates in all four quadrants.

## Level 6

Pupils order and approximate decimals when solving numerical problems and equations [for example, x $3+x=20$ ], using trialandimprovement methods. Pupils are aware of which number to consider as 100 per cent, or a whole, in problems involving comparisons, and use this to evaluate one number as a fraction or percentage of another. They understand and use the equivalences between fractions, decimals and percentages, and calculate using ratios in appropriate situations. They add and subtract fractions by writing them with a common denominator. When exploring number sequences, pupils find and describe in words the rule for the next term or nth term of a sequence where the rule is linear. They formulate and solve linear equations with wholenumber coefficients. They represent mappings expressed algebraically, and use Cartesian coordinates for graphical representation interpreting general features.

## Level 7

In making estimates, pupils round to one significant figure and multiply and divide mentally. They understand the effects of multiplying and dividing by numbers between 0 and 1. Pupils solve numerical problems involving multiplication and division with numbers of any size, using a calculator efficiently and appropriately. They understand and use proportional changes, calculating the result of any proportional change using only multiplicative methods. Pupils find and describe in symbols the next term or nth term of a sequence where the rule is quadratic; they multiply two expressions of the form $(x+n)$; they simplify the corresponding quadratic expressions. Pupils use algebraic and graphical methods to solve simultaneous linear equations in two variables. They solve simple inequalities.

## Level 8

Pupils solve problems involving calculating with powers, roots and numbers expressed in standard form, checking for correct order of magnitude. They choose to use fractions or percentages to solve problems involving repeated proportional changes or the calculation of the original quantity given the result of a proportional change. They evaluate algebraic formulae, substituting fractions, decimals and negative numbers. They calculate one variable, given the others, in formulae such as $\mathrm{V}=\mathrm{Yr} 2 \mathrm{~h}$. Pupils manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions. They know that a $2-\mathrm{b} 2=$ $(a+b)(a-b)$. They solve inequalities in two variables. Pupils sketch and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations.

## Exceptional Performance

Pupils understand and use rational and irrational numbers. They determine the bounds of intervals. Pupils understand and use direct and inverse proportion. In simplifying algebraic expressions, they use rules of indices for negative and fractional values. In finding formulae that approximately connect data, pupils express general laws in symbolic form. They solve simultaneous equations in two variables where one equation is linear and the other is quadratic. They solve problems using intersections and gradients of graphs.

## Phase2: Application of framework to collect data

Collection of a mass of items from International National Curricula within framework Contributers:

- Partner 1 - Comenius University, Bratislava, Slovakia (coord. Ivan Kalas)
- Partner 2 - Knowledge Lab, London, UK (coord. Celia Hoyles and Richard Noss)
- Partner 3 - Cnotinfor, Portugal (coord. Secundino Correia)
- Partner 4 - EL University, Budapset, Hungary (coord. Marta Turcsanyi)
- Partner 5 - Centre for New Technologies Research in Education (CeNTRE), University of Warwick, UK (coord. Dave Pratt)

The collection has been filtered and common elements identified for further processing. We however include a small collection of internet sites, where relevant material can be accessed.

Mathematical internet portals and sites


## Phase3: Analysis of items for common components of international relevance

We studied and compared potential of building computer environments in which children can explore and discover basic mathematics concepts and relations. We studied all partners' national mathematics curricula for children aged 8 to 12 and identify examples of the common building blocks out of which current activities (pages or "screens") could be constructed.

This report contains a summary of visual interactive building blocks that we found to be accessible for applying into various national mathematics curricula for children aged 8 to 12. Levels indicate those identified in the framework. Items have been grouped by topics.

## 3D shapes



Visualizing 3D shapes from 2D drawings.
Thinking about visual elements including form and space.
(Making drawings, getting experience by changing the viewpoint on a screen )


## Animations

## Animated shapes

- Cut out copies of these grids to make a flick book.
- Can you make the triangle spin around the ring?

level 6.

Multimedia shapes

level 6.

level 6.

Recognizing a shape which has been transformed or rotated.
(Getting own experience with making such transformations.)

- Using graphic SW, like LogoMotion, Paint, Paint Shop Pro, Corel Draw
$\square$


## Bigger or smaller

- calculate a given fraction of a given quantity [for example, for scale drawings and construction of models, down payments, discounts], expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal
- understand and use unit fractions as multiplicative inverses [for example, by thinking of multiplication by one-fifth as division by 5 , or multiplication by sixsevenths as multiplication by 6 followed by division by 7 (or vice versa)] ; multiply and divide a fraction by an integer, and multiply a fraction by a unit fraction
- convert simple fractions of a whole to percentages of the whole and vice versa [for example, analysing diets, budgets or the costs of running, maintaining and owning a car] ,then understand the multiplicative nature of percentages as operators [for example, 30\% increase on \#150 gives a total calculated as \#(1.3 * 150) while a $20 \%$ discount gives a total calculated as \#(0.8 * 150)]
- divide a quantity in a given ratio [for example, share \#15 in the ratio of 1:2]



Solve problems, recognize simple patterns and relationships, generalize and predict. Suggestions like "what if..."


## Charts, pie-charts and graphs

- use the conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for $m$ and $c$ ) that equations of the form $y=m x+c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which $y$ is given explicitly in terms of $x$ [for example, $y=2 x+3$ ], or implicitly [for example, $x+y=7$ ]
- construct linear functions from real-life problems and plot their corresponding graphs; discuss and interpret graphs arising from real situations; understand that the point of intersection of two different lines in the same two variables that simultaneously describe a real situation is the solution to the simultaneous equations represented by the lines; draw line of best fit through a set of linearly related points and find its equation


Analyze evidence and drawing conclusion. (Using known data, geographical data...)


## Coordinates, coordinate-systems

- use the conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for $m$ and $c$ ) that equations of the form $y=m x+c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which $y$ is given explicitly in terms of $x$ [for example, $y=2 x+3$ ], or implicitly [for example, $x+y=7$ ]

Give the co-ordinates
s map records finds at an archeological dig.


Record the co-ordinotes of the finds in this table.

| find | co-ordinates |
| :--- | :--- |
| brooch |  |
| sword |  |
| helmet |  |
| fireplace |  |
| pottery |  |
| arrow head <br> burial chamber <br> necklace |  |


level 4.
level 6.


To use and draw maps, plans at a range of scale. (Finding places in maps, plans, coordinates...)

## Battleships games, treasure hunts



## Counting

## Counting

a) count reliably up to 20 objects at first and recognise that if the objects are rearranged the number stays the same; be familiar with the numbers 11 to 20 ; gradually extend counting to 100 and beyond

## Number patterns and sequences

b) create and describe number patterns; explore and record patterns related to addition and subtraction, and then patterns of multiples of 2,5 and 10 explaining the patterns and using them to make predictions; recognise sequences, including odd and even numbers to 30 then beyond; recognise the relationship between halving and doubling

## The number system

c) read and write numbers to 20 at first and then to 100 or beyond; understand and use the vocabulary of comparing and ordering these numbers; recognise that the position of a digit gives its value and know what each digit represents, including zero as a placeholder; order a set of one- and two-digit numbers and position them on a number line and hundredsquare; round any two-digit number to the nearest 10 .
d) use a calculator for calculations involving several digits, including decimals; use a calculator to solve number problems [for example, 4 ? * $7=343$ ] ; know how to enter and interpret money calculations and fractions; know how to select the correct key sequence for calculations with more than one operation [for example, 56 * (87-48)]

|  | Num | k | ac |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | T | U |  |
|  | 0 |  | 1 | one |
|  | *980emee | 1 | 0 | ten |
|  | ****eeee | 1 | 1 |  |
|  | **** |  |  |  |
|  | *eseeneeesco |  |  |  |
|  | level 1. |  |  |  |



Making some tests. Getting own experiment. Trying things out, using visual ways to see the outcome.


## Fractions

- understand unit fractions [for example, one-third or one-eighth] then fractions that are several parts of one whole [for example, two-thirds or five-eighths] , locate them on a number line and use them to find fractions of shapes and quantities
- understand simple equivalent fractions and simplify fractions by cancelling common factors; compare and order simple fractions by converting them to fractions with a common denominator, explaining their methods and reasoning
- recognise the equivalence between the decimal and fraction forms of one half, quarters, tenths and hundredths; understand that 'percentage' means the 'number of parts per 100' and that it can be used for comparisons; find percentages of whole number quantities, using a calculator where appropriate
- recognise approximate proportions of a whole and use simple fractions and percentages to describe them, explaining their methods and reasoning
- solve simple problems involving ratio and direct proportion
- use fraction notation; understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator
- use ratio notation, including reduction to its simplest form and its various links to fraction notation
- recognise where fractions or percentages are needed to compare proportions; identify problems that call for proportional reasoning, and choose the correct numbers to take as $100 \%$, or as a whole.
- understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of' [for example, 10\% means 10 parts per 100 and $15 \%$ of $Y$ means 15 onehundredths * Y] ; use percentage in real-life situations [for example, commerce and business, including rate of inflation, VAT and interest rates]


|  | level 6. |
| :---: | :---: |
| Make the numerator and denominotor as small as you can. <br> What fraction of the totol moss is each kind of nut? <br> Make up some other problems like these. <br> level 6. | You can solve problems by finding percentoges of quantities. <br> How much do you pay for each part-roll of wallpaper? <br> level 6. |
| Fraction flags <br> level 3. | Fraction flags <br> - Shode the squares to design different flogs, using two colours. Record the fractions as shown under the first flog. <br> level 3. |
|  |  |



## Making one whole

| <ely | Two halves or four quarters makes one white. |
| :--- | :--- |
| idea |  |

Match the halves and quarters to make whole cheeses Write the letters that make each whole cheese like this:
a and ir

1 and $\frac{1}{2}$ make I whole


11

level 6.

## Discount fares

- Bus and troin romprnies ntter discounts an standord aras Campete these farz tables

London return, full fare $£ 100$

| fare type | dtscourt | saving | fare |
| :---: | :---: | :---: | :---: |
| Standara |  |  | $£ 00$ |
| Your. jerson | $50 \%$ | $£ 50$ | $£ 50$ |
| CAP | $25 \%$ | $£ 25$ |  |
| Weekend | $0 \%$ |  |  |
| Early booking | $20 \%$ |  |  |
| Las: minute | $75 \%$ |  |  |
| Famil_ nil card | $40 \%$ |  |  |



Paris return, full fare $E 200$

| fare type | discount | saving | fare |
| :---: | :---: | :---: | :---: |
| Stancard | - | - | $£ 200$ |
| Young prsonn | $50 \%$ | $£ 00$ |  |
| OAP | $25 \%$ | $£ 25$ |  |
| Weckend | $10 \%$ |  |  |
| Early booking | $20 \%$ |  |  |
| Las: mininie | $75 \%$ |  |  |
| Eamily rail tard | $40 \%$ |  |  |

level 5.
level 6.

## Unfinished buildings

1. tow manir borks ore missitg from these temple ruirs?

cuboid
missing blocks $\qquad$

mssing blocks $\square$
2. cuboid missing blocks
level 4.
level 5.
Understanding what fractions are.
Make and justifying estimate of numbers and proportions.
Fractions microworld!



## Games with numbers




level 2.

level 3.

Mathematical crossword
Across

1. Next prime number after 5 (5)
2. Most common member of a sat of data (4)
3. Third triangular nuraber (3)
4. Three squared (4)
5. Horizontal line of celt on a grid (3)
6. The decimal number (3)

## Down

1. The result of adding (3)
2. A number divisible by 2 (4)
3. Has length but no width (4)
4. Greek eight (4)
5. 2-D plan for a 3-D shape (3)

level 6.
Make a pattern

level 2.

" 푼․․․․․․․․․․․․

1


Draw bow-ties on $\frac{1}{3}$ of the toys.
Draw hats on $\frac{2}{3}$ of the toys.
$\frac{2}{3}$ of the toys are holding
ice-creams. Draw hem.
$\frac{1}{3}$ of the tons have ice -creams with of lake. D ow them.
$\frac{2}{3}$ of the toys have red noses. Colour them.


Games where knowledge of mathematical facts is needed.
Maze games, puzzle games, dot to do pictures, coloring exercises, tangramm games



## Geometry, angles

- set up simple equations [for example, find the angle $a$ in a triangle with angles $a, a+10, a+20$ ] ; solve simple equations [for example, $5 x=7$, $3(2 x+1)=8,2(1-x)=6(2+x), 4 x^{2}=36,3=z$ ], by using inverse operations or by transforming both sides in the same way
- use formulae from mathematics and other subjects [for example, formulae for the area of a triangle, the area enclosed by a circle, density = mass/volume] ; substitute numbers into a formula; derive a formula and change its subject [for example, convert temperatures between degrees Fahrenheit and degrees Celsius, find the perimeter of a rectangle given its area A and the length I of one side]



## Numbers in life

- activities focused on the major ideas of statistics, including using appropriate populations and representative samples, using different measurement scales, using probability as a measure of uncertainty, using randomness and variability, reducing bias in sampling and measuring, and using inference to make decisions


## Secret codes



## osectivesDistance table

## Numeracy

Solve problems in 'real life' involving nensures.
Round any wo-digit number to the nearest,
10 ard ame chree-digit number to the rearest $10 C$.
Find te position of a square on a grid of squares with the rows and columns labeled.
Geography
Use app-opriate geographical vacabulary. Use maps did plais at a I dige of scales. Sudy 1 ange of places including the Urited Kirgdom.
Lirks to QCA Geography Unit 6:
livestigating our local area.

level 3.
lis map shows the city bus routes.
ie distances are in kilometres.
Factory $\left\{\begin{array}{l}\text { Docks } \\ 6 \\ \text { Stadium } \\ \text { Square } \\ \text { Church Airport }\end{array}\right.$
${ }_{4}$ Library $^{3} \begin{gathered}18 \text { is } 2 \\ \text { station } \\ 5\end{gathered}{ }^{3}$ Station $^{7}$
level 3.

The monster has a recipe for 4
He wants to make enough for 1 .
Find $\frac{1}{4}$ of each item.

level 2.

## How many legs?


level 2.

## Internet search



Incroduce the lesson by listing some quadrilateral words on the board: quadidateral
parallelogram
thombus
trapezium
Explain that the children's task during this lesson is to use the Internet to resea the meanings of these words, and to writ brief report on their findings including diagrams and definitions.

Set the children to work in pairs ors groups at computers. Discuss the use of Interner search engines and strategies for searching productively for the informatio required. If children have not done so bef demonstiate how to copy text and diagr from web pages for pasting and editing in their personal word-processed repors.
level 6.

level 6.

level 6.
( The planets

Sequence the planets in the blank fables:

| planets in <br> alphabetical <br> order | distance <br> from Sun <br> in millions <br> of $\mathbf{~ k m}$ | size <br> in $\mathbf{~ k m}$ | surface <br> temperature <br> in ${ }^{\circ} \mathrm{C}$ |
| :--- | :---: | :---: | :---: |
| Earth | 149.6 | 12756 | 55 |
| Jupiter | 778.3 | 142800 | -160 |
| Mars | 227.9 | 6794 | 25 |
| Mercury | 59.9 | 4878 | 430 |
| Neptune | 4496.7 | 49500 | -213 |
| Pluta | 5900 | 2284 | -230 |
| Saturn | 1427.0 | 120000 | -150 |
| Uranus | $2866^{\circ} .6$ | 51800 | -220 |
| Venus | 108.2 | 12104 | 480 |



| tianats in <br> tistance <br> toder <br> sending) | distance <br> from Sun <br> in millions <br> of km |
| :--- | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |


level 5.

Solving lifelike problems, problems from real life, where mathematical knowledge is needed.
Counting prizes, measuring distance on maps, grouping thing...


## Logo



Forward 10 Right 90 Forward IO Right 90 Forward 10 Right 90 Forward 10
 Right 90
level 4.

Understanding relative directions, telling someone how to go somewhere in his view. Ordering angles, understanding them, drawing polygons...
Estimating distances.


## Money



level 1.
How many do we need?

-or much is each cake?
How nany cakes are there in 3 packs?

Hyw many packs do you need : qive one cake each to 30 thildren? $\qquad$
two nuch would they cost thogether? $\qquad$


How much is each can? $\qquad$
Four children share a pack.
How many cans do they have each?
How many cans are there in 2 packs? $\qquad$
How much do
2 packs cost? $\qquad$

tou many pencils are there altogether in the carton?
Thenty children share all che pencils berween them. How many to they each have? $\qquad$
Tie pencils cost $10 p$ each. How much is a pack of ten? $\qquad$
How much is a whole carton?
level 3.

## Discount fares

- Bus and train rompranies ntter discoi.nts an standord ares Comp ete these fare tables
London return, full fare $£ 100$

| fare type | discount | saving | fare |
| :---: | :---: | :---: | :---: |
| Standari |  |  | $£ 00$ |
| Your. oerson | $50 \%$ | $£ 50$ | $£ 50$ |
| CAP | $25 \%$ | $£ 25$ |  |
| Weekend | $0 \%$ |  |  |
| Early booking | $20 \%$ |  |  |
| Las: minuse | $75 \%$ |  |  |
| Famil_ nil card | $40 \%$ |  |  |



Paris return, full fare $£ 200$

| fare type | discount | saving | fare |
| :---: | :---: | :---: | :---: |
| Stancara | - | - | $£ 200$ |
| Young prson | $50 \%$ | $£ 00$ |  |
| OAP | $25 \%$ | $£ 25$ |  |
| Weekend | $10 \%$ |  |  |
| Early booking | $20 \%$ |  |  |
| Las: minuer | $75 \%$ |  |  |
| Zumily rail card | $40 \%$ |  |  |

level 5.
level 6.

level 1.
From recognizing coins, values of coins, solving problems involving money... to counting prizes, VAT, sale prizes...


## Music and math's



From making rhythms, till understanding the mathematical context of music...
Making own experiments.


## Number lines

- read and write numbers to 20 at first and then to 100 or beyond; understand and use the vocabulary of comparing and ordering these numbers; recognise that the position of a digit gives its value and know what each digit represents, including zero as a place-holder; order a set of one and two-digit numbers and position them on a number line and hundred-square; round any two-digit number to the nearest 10.
- solve simple linear inequalities in one variable, and represent the solution set on a number line
- use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use positive numbers, both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple and common factor

- These frogs jump in halves. Continue the Ilnes to show where the Irogs land.

- Draw an arrow to show half of the number in the clicle.

level 4. - estimate

Number lines help to visualize bigger/smaller.
Recognizing contents, values of numbers, estimate values. Visualizing the estimated values.


## Operating with numbers

## Number operations and the relationships between them

a) understand addition and use related vocabulary; recognise that addition can be done in any order; understand subtraction as both 'take away' and 'difference' and use the related vocabulary; recognise that subtraction is the inverse of addition; give the subtraction corresponding to an addition and vice versa; use the symbol ' $=$ ' to represent equality; solve simple missing number problems [for example, $6=2+$ ? ]
b) understand multiplication as repeated addition; understand that halving is the inverse of doubling and find one half and one quarter of shapes and small numbers of objects; begin to understand division as grouping (repeated subtraction); use vocabulary associated with multiplication and division

## Mental methods

c) develop rapid recall of number facts: know addition and subtraction facts to 10 and use these to derive facts with totals to 20, know multiplication facts for the *2 and *10 multiplication tables and derive corresponding division facts, know doubles of numbers to 10 and halves of even numbers to 20
d) develop a range of mental methods for finding, from known facts, those that they cannot recall, including adding 10 to any single-digit number, then adding and subtracting a multiple of 10 to or from a two-digit number; develop a variety of methods for adding and subtracting, including making use of the facts that addition can be done in any order and that subtraction is the inverse of addition
e) carry out simple calculations of the form $40+30=?, 40+?=100,56 ?=10$; record calculations in a number sentence, using the symbols + , , , division and $=$ correctly [for example, $7+2=9$ ] .

Count on and back in tens or hundreds from any two- or three-digit number; recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back

- develop further their understanding of the four number operations and the relationships between them including inverses; use the related vocabulary; choose suitable number operations to solve a given problem, and recognise similar problems to which they apply
- find remainders after division, then express a quotient as a fraction or decimal; round up or down after division, depending on the context
- understand the use of brackets to determine the order of operations; understand why the commutative, associative and distributive laws apply to addition and multiplication and how they can be used to do mental and written calculations more efficiently
- use calculators effectively and efficiently: know how to enter complex calculations using brackets [for example, for negative numbers, or the division of more than
one term], know how to enter a range of calculations, including those involving measures [for example, time calculations in which fractions of an hour need to be entered as fractions or decimals]
- use the function keys for reciprocals, squares, square roots, powers, fractions (and how to enter a fraction as a decimal); use the constant key
- understand the calculator display, interpreting it correctly [for example, in money calculations, and when the display has been rounded by the calculator], and knowing not to round during the intermediate steps of a calculation.

| $\left\|\begin{array}{c} 3+4=7 \\ \because \\ 4+8=\Delta \mid 6+\Delta=9 \end{array}\right\|$ level 2. | Missing digits and signs <br> A spider has blotted the ink! Write these number sentences correctly. <br> $6.295+\square=305$ <br> $7.97 \square 69=28$ <br> 8.7 $\square$ $4=28$ <br> १. <br> 11. $\square \div 2=80$ <br> 12. 12 $\square$ $9=108$ <br> 13. $100 \div$ $\square$ $=25$ <br> level 4. |  |  |
| :---: | :---: | :---: | :---: |
| 345 273 <br> $\frac{29}{14}$ +16 <br> 60 21 <br> $\frac{300}{374}$ -12$\begin{aligned} 53=50+3= & 40+13 \\ \underline{-27}=\underline{20+7}= & 20+7 \\ & 20+6=2 \end{aligned}$ <br> level 3. |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
| How many in a pack? <br> level 3. |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

## Arithmetic



Remember, when you multiply or divide an inequality by a negative number, you must reverse the inequality symbol.

Click on each equation to see how it is solved.

$$
\begin{array}{ll}
2 x-12=5 & \begin{array}{l}
67=11 n+1 \\
67-1=11 n+1-1 \text { step } 1 \\
\\
67=11 n+1 \\
\\
\\
7=4-2 y \\
\\
\\
\\
\\
\\
\hline 66=11 n \\
\frac{66}{11}=\frac{11 n}{11} \text { step } 2 \\
6=n \quad \text { solution } \\
-5 x+3 x+1 \geq-14
\end{array} \\
\begin{array}{l}
n=6 \quad \text { (it's standard to put the variable on th } \\
\text { left side.) }
\end{array} \\
\text { Previous }
\end{array}
$$



## Pin board




## Probability

Activities focused on the major ideas of statistics, including using appropriate populations and representative samples, using different measurement scales, using probability as a measure of uncertainty, using randomness and variability, reducing bias in sampling and measuring, and using inference to make decisions


Making own tries. Tests, making simulations, and numerous experiments, to see the resolute...
Discuss the chance or likelihood of particular events. Learn and analyze evidence and draw conclusions.
Click on the speaking dice

## Sequences

Recognise and describe number patterns, including two- and three-digit multiples of 2,5 or 10 , recognising their patterns and using these to make predictions; make general statements, using words to describe a functional relationship, and test these; recognise prime numbers to 20 and square numbers up to 10 * 10 ; find factor pairs and all the prime factors of any two-digit integer

- generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers)
- find the first terms of a sequence given a rule arising naturally from a context [for example, the number of ways of paying in pence using only $1 p$ and $2 p$ coins, or from a regularly increasing spatial pattern] ; find the rule (and express it in words) for the $n$th term of a sequence
- generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the $n$th term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated


level 2.


## One in every...

- Fill in the missing numbers.


1. There are $\qquad$ tortoises for every rabbi-:
2. One in evert, $\qquad$ animals is a rabbit.

3. There are $\qquad$ birds for every rabbit.
4. One in every $\qquad$ animals is a frog.
5. One in every $\qquad$ animals is a abbit.
6. Two in every $\qquad$ animals are birds.
level 4.

Making patterns, visual sequences, than number sequences...



## Shapes

Explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further and understanding the importance of a counter-example; identify exceptional cases when solving problems; make conjectures and check them for new cases.


Learning the shape...
Learning to recognize different shapes, the name them, see the difference...

|  |  |
| :---: | :---: |
| http://www.coolmath4kids.com/ | $3,6,3,6$ <br> http://www.coolmath4kids.com/ |

## Symmetry

- explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further and understanding the importance of a counter-example; identify exceptional cases when solving problems; make conjectures and check them for new cases
- explore, identify, and use pattern and symmetry in algebraic contexts [for example, using simple codes that substitute numbers for letters] , investigating whether particular cases can be generalised further, and understanding the importance of a counter-example; identify exceptional cases when solving problems



Making experiments, playing games.


## Temperature



Measuring, and understanding different temperatures.


Time

level 2.
level 3.

level 1.
level 4.


From learning to know the time, and use clocks and watches till the using timelines like number-lines...


## Words for math's

## Problem solving

a) approach problems involving number, and data presented in a variety of forms, in order to identify what they need to do
b) develop flexible approaches to problem solving and look for ways to overcome difficulties
c) make decisions about which operations and problemsolving strategies to use
d) organise and check their work

Communicating
e) use the correct language, symbols and vocabulary associated with number and data
f) communicate in spoken, pictorial and written form, at first using informal language and recording, then mathematical language and symbols



Learning the words for mathematical things, numbers, expressions...


## Evaluation

## General comments

In certain areas of mathematics education - researchers have managed to build dynamic visual environments with a high level of interaction and visualization. Learners can formulate conjectures in such environments, test and modify them, building their own understandings step-by-step, they concretize their mathematics concepts into meaningful, richly-connected entities.

However most of the indicated environments are closed and their variability is very limited both to educators and students.

These theme ideas could be utilised in a more variable way if they would be open for manipulations and further variations. Furthermore, by providing flexible toy-like objects living in these virtual environments, that obeys the laws of their meaningful functionality, motivation of children would rise nearer to that of playful experiences and allow them to learn important concepts while being involved in a virtual tangible environment.

## Documenting and analysing curricula from the developer's viewpoint

When analyzing partners' national mathematics curricula and school textbooks for children aged 8 to 12, we have had several interrelated goals in our minds:

- the main goal of the Trail 1, which is to identify and select concepts, common to different European mathematics education approaches and appropriate for being modelled as interactive building blocks,
- iteratively develop a kind of framework for their classification, representation and description as good candidates for general building blocks, useful and reusable in all or most of the approaches in elementary mathematics education,
- iteratively develop a framework for their prototyping, generalizing our previous experience with developing visual fractions Logo microworld within the Minerva CoLabs project.

Main observations of the CUB partner (here in the role of the developers of interactive interfaces for children to support their learning processes) within the Trail 1 are:

- all national curricula, math textbooks and other educational materials devoted to this age group are very similar (which supports our strong believe that common building blocks can be identified and prototype in a way useful for different teaching/learning approaches - this assumption will be further investigated in Trails 3 and 4),
- if there are any differences, they are most often (slightly) different sequencing of the topics, (slightly) different prioritizing, giving more space to certain concepts, techniques and skills,
- all educational materials try to be highly visual, colourful, amusing, well structured and balanced, well organized - from the "presentational" point of view as well as from the educational point of view,
- learning materials try to use different graphical items, colours, fonts, marks... etc. to increase the motivation,
- authors of the learning materials try to develop and apply certain "visual schemes" with the goal to facilitate learning processes. Identifying these schemes is very important for our further work within the Trail 2 (which is prototyping visual interactive building blocks as visual programmable objects) because they are closely interconnected with the way how the key mathematics concepts are presented and explored. In a way, these are different styles or settings for presenting concepts, relations, techniques etc. For example, a small squared coloured in red is always used at the place where children are supposed to insert some missing symbol, relation, value, object... etc. We think that these "settings" or styles of presentation are inseparable from the general visual objects or blocks,
- all learning materials tend to use as many as possible examples from real life. They present concepts and relations using cars, trains, boats, balloons, candies... etc. All of these objects however lack at least one part of the goal why they have been used - they are static, not moving, not "developing", "building" etc.
- all educational materials try to present one concept within the environment of several other related concepts - already familiar to children. For example, when percentages are presented, they are immediately related to decimal fractions and/or ratios etc. Obviously, the goal is to give children complex picture of different representations of identical or related concepts,
- educational materials try (usually with modest efficiency) to present processes, sequences of actions, steps to be taken, actions, dynamic behaviours.
- As an example of such process we can find activities like:
- Position the frog on the number line at the value of 2.5 ,
- Taken that the frog can jump only by halves and it jumps now from left to the right, let it takes five jumps,
- Where on the number line the frog will end up?
- Such activities are extremely useful and important constituent of the whole learning process, however lack (at least partly) the main reason why they are being used - the dynamism which cannot be really exploited within static media,
- all educational materials are limited in the way how collaborative learning can be involved and exploited. They most usually address an individual learner. Also, the framework of the materials gives very limited space for dynamic, interactive explorations, learning by doing, learning by exploring - individually or within a group of learners.
Building on all of these observations we want to prototype several of mostly often used visual representations and "styles of presentation" to reach the goal of the Trail

2, which is Prototyping visual interactive building blocks as visual programmable objects. We plan to specify and prototype a kind of consistent design and functionality of visual building blocks so that the resulting format will offer high level of interactivity and flexibility.

## Aftermath

All building blocks will then be prototyped and instantiated as interactive, manipulable and visual Imagine objects, which may - through their interactive features - support and enhance the process of learning. We will then examine how these prototypes could be used to author computer environments in which children learn about mathematics concepts and relations.

